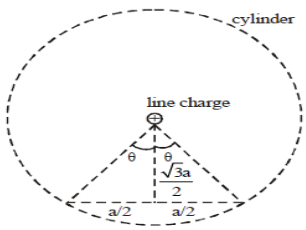
1. Ans. 6



$\theta = \tan^{-1} (a/2 / \sqrt{3}a/2) = \tan^{-1} (1/\sqrt{3}) = 30^0$
Electric flux passing through the whole cylinder
$= Q_{in} / \varepsilon_0 = \lambda L / \varepsilon_0$
Flux through the rectangular surface ABCD
$=(60/360) \times \lambda L / \varepsilon_0$
$= \lambda L / 6\varepsilon_0$
So, n=6

2. Ans. 2
KE of the ejected electron = Energy of incident photon – energy required to ionize the electron from nth orbit
10.4 = 1242/90 – | En |
= $1242/90 - 13.6/n^2$
Solving this equation gives
n= 2

3. Ans. 2
At height h
$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$
g' = g/4 (Given)
$g/4 = \frac{g}{\left(1+\frac{h}{R}\right)^2}$
➔h=R
Since decrease in KE = increase in PE
$1/2mv^2 = mgh/(1+h/R)$
➔$v^2/2 = gh/(1+h/R) = 1/2gR$
➔$v = \sqrt{gR}$
$v_{esc} = \sqrt{2gR} = v\sqrt{2}$
n = 2

4. Ans. 7
In case of pure rolling, ME remains constant
$\frac{\text{Trlational KE}}{\text{Rotational KE}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2} = \frac{mv^2}{\frac{1}{2}mR^2\left(\frac{v}{R}\right)^2} = 2$.

Trlational KE(K_t) = 2/3

Total KE= $3/2K_t$ = ¾ mv^2

Applying conservation of energy ,

$\frac{3}{4}m\,3^2 + mg\,30 = \frac{3}{4}m\,(V2)^2 + mg(27).$

Solving this gives

V_2 = 7 m/s

5. Ans. 2

p$\propto \frac{r^2}{\lambda^4}$ (For Spherical Stars)

⇨ $10^4 = (400)^4 \text{ X } \left(\frac{\lambda b}{\lambda a}\right)^4$

⇨ $\frac{\lambda a}{\lambda b} = 2$

6. Ans. 3

At t = 0

$P_{\text{required}} = P_{\text{availaable}}$ x (12.5/100) = $P_{ava}/8$

$P_{ava} = 8P_{req}$

After first half life $P_{ava} = 4P_{req}$

After 2nd half life $P_{ava} = 2P_{req}$

After 3rd half life $P_{ava} = P_{req}$

Hence Power required by the village can be met upto 3 half lives.

N = 3

7. Ans. 3

For maxima

Path difference = nλ

$\sqrt{d^2 + x^2}\left(\frac{4}{3} - 1\right) = n\lambda$

⇨ $x^2 = 9n^2\lambda^2 - d^2$

⇨ $p = 3$

8. Ans. 7

Image by mirror is formed at 30 cm from mirror at its right and finally by the combination it is formed at 20 cm on right of the lens. So in air medium, magnification by lens is unity.

In second medium μ = 7/6, focal length of lens

$\frac{\frac{1}{10}}{\frac{1}{f}} = \frac{(1.5-1)\left(\frac{1}{R1} - \frac{1}{R2}\right)}{\left(\frac{1.5}{\frac{7}{6}} - 1\right)\left(\frac{1}{R1} - \frac{1}{R2}\right)}.$

$f = \frac{35}{2}\,cm.$

So in second medium, final image is formed at 140 cm to the right of the lens.

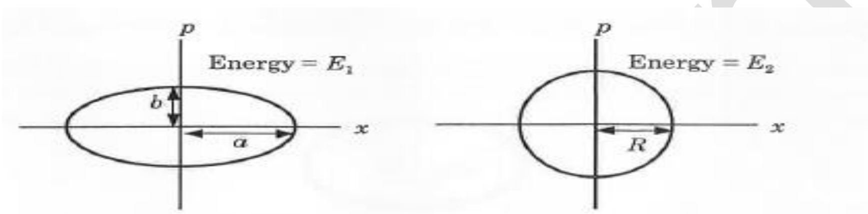
Second medium does not change the magnification by mirror.

$\left|\frac{M2}{M1}\right|$ = | $M_{m2}M_{l2}$ / $M_{m1}M_{l1}$ | =7

9. Ans. B , C
For vernier calipers,
1 MSD = 1/8 cm
1 VSD = 1/10 cm
For screw guage,
Pitch (p) = 2 MSD
So least count = p/100
➔Option B and C are correct

10. Ans. A, C, D
$h = [ML^2T^{-1}], c = [LT^{-1}], G = [M^{-1}L^3T^{-2}].$
$M \propto \sqrt{\frac{hc}{G}}, L \propto \sqrt{\frac{hG}{c^3}}.$

11. Ans. B, D



From figure
$E1 = \frac{m(\omega 1)^2}{2} a^2$
$E2 = \frac{m(\omega 2)^2}{2} R^2$
$\frac{E1}{E2} = \left(\frac{\omega 1}{\omega 2}\right)^2 \left(\frac{a}{R}\right)^2 = \left(\frac{\omega 1}{\omega 2}\right)^2 n^2$ -------1
$Also\ \frac{E1}{E2} = \left(\frac{p1}{p2}\right)^2 = \left(\frac{b}{R}\right)^2$
$Or\ \frac{E1}{E2} = \frac{1}{n^2}$ -------2
Using 1 and 2
$\left(\frac{\omega 1}{\omega 2}\right)^2 = \frac{1}{n^4}$
$\frac{\omega 1}{\omega 2} = \frac{1}{n^2}$
$\frac{\omega 2}{\omega 1} = n^2$
$\frac{E1}{E2} = \frac{\omega 1}{\omega 2}$
$\frac{E1}{\omega 1} = \frac{E2}{\omega 2}$

12. Ans. D

Using conservation of angular momentum

We get

$$MR^2\omega = \left(MR^2 + \frac{M}{8}X\frac{9}{25}R^2 + \frac{M}{8}x^2\right)X\frac{8}{9}\omega$$

$$\frac{9}{8}R^2 = R^2 + \frac{9}{200}R^2 + \frac{x^2}{8}$$

$$x^2 = \frac{16}{25}R^2$$

$$x = \frac{4}{5}R$$

13. Ans. C

Case I:

$$F = \frac{\lambda q}{2\pi\varepsilon o(r+x)}\, i + \frac{\lambda q}{2\pi\varepsilon o(r-x)}\,(-i)$$

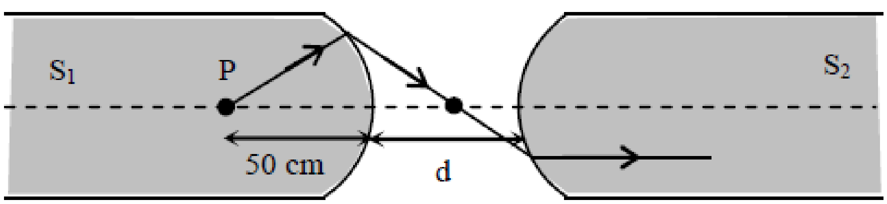
$$= \frac{\lambda q}{2\pi\varepsilon o r^2}x\,(-i)$$

So +q charge will perform SHM with time period T = $2\pi\sqrt{\frac{\pi r^2\varepsilon o m}{\lambda q}}$

Case II :

Resultant force will act along the direction of displacement

14. Ans. B



For 1st refraction

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$
$$=> v = 50cm$$

For 2nd refraction

$$\frac{1.5}{\infty} - \frac{1}{-x} = \frac{1.5-1}{+10}$$
$$=> x = 20$$

So d = 20+50 = 70 cm

15. Ans. A, B, C

We know,

$.Fm = i(dl\ X\ B)$

In option A

$Fm \propto (L + R)$, If B is along z axis

In option B

$Fm = 0$, If B is along x axis.

In option C

$Fm \propto (L + R)$, If B is along y axis

16. Ans. A, B, D

$$\frac{Average\ energy}{mole} = \frac{\frac{3}{2}RT+\frac{5}{2}RT}{2} = 2RT.$$

Option B)

$$\frac{Speed\ of\ sound\ in\ mixture}{Speed\ of\ sound\ in\ Helium\ gas} = \sqrt{\frac{\Upsilon mix}{Mmix}\ X\ \frac{Mhe}{\Upsilon he}}$$

$$\Upsilon mix = \frac{\sum NiCp}{\sum NiCv}$$

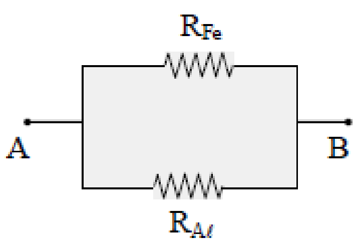
Average molar mass of mixture $=\frac{1}{2}X4 + \frac{1}{2}X2 = 2 + 1 = 3$

$$= \sqrt{\left(\frac{\frac{3}{2}}{3}\right)\left(\frac{4}{\frac{5}{3}}\right)} = \sqrt{\frac{6}{5}}$$

Option D

$$Vrms \propto \frac{1}{\sqrt{M}} \frac{(Vrms)Helium}{(Vrms)Hydrogen} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

17. Ans. B



$R_{fe} = \frac{\rho\ X\ 50\ X\ 10^{-3}}{(2X10^{-3})^2} = 1250\ ohm$

$R_{al} = \frac{\rho\ X\ 50\ X\ 10^{-3}}{(49-4)X10^{-6}} = 30\ ohm$

R_{eq} = 1250 || 30 ohm

$= \frac{1250X30}{1250+30} = \frac{1875}{64} X\ 10^{-6}\ ohm$

18. Ans. A, C

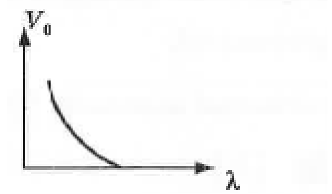
By photoelectric effect equation,

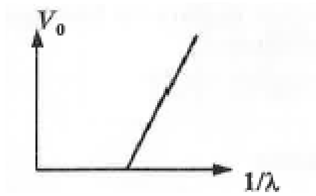
$hf - Wo = eVo$

$h\frac{c}{\lambda} - Wo = eVo$

⇨ $\frac{1}{\lambda} \propto Vo$

So following graphs are correct.





19. Ans.

(A) - (R, T);
(B) - (P, S);
(C) - (P, Q, R, T);
(D) - (P, Q, R, T)

Theory based question

20. Ans.

(A) - (P, Q, R, T);
(B) - (Q, S);
(C) - (P, Q, R, S);
(D) - (P, R, T)

For part A

$$F(x) = -\frac{du}{dx} = \frac{2Uox}{a^2}\left(1 - \frac{x^2}{a^2}\right)$$

For part B

$$F(x) = -\frac{du}{dx} = \frac{-Uo}{2}\left(\frac{2x}{a^2}\right)$$

For part C

$$F(x) = -\frac{du}{dx} = -\frac{Uo}{2}\left[\frac{2x}{a^2}e^{-\frac{x^2}{a^2}} + \frac{x^2}{a^2}Xe^{-\frac{x^2}{a^2}}X\frac{-2x}{a^2}\right]$$

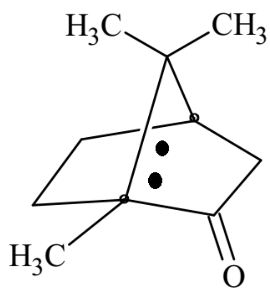
$$F = \frac{2Uo}{a^2}e^{-\frac{x^2}{a^2}}(1 - \frac{x^2}{a^2})$$

$$F$$

For part D

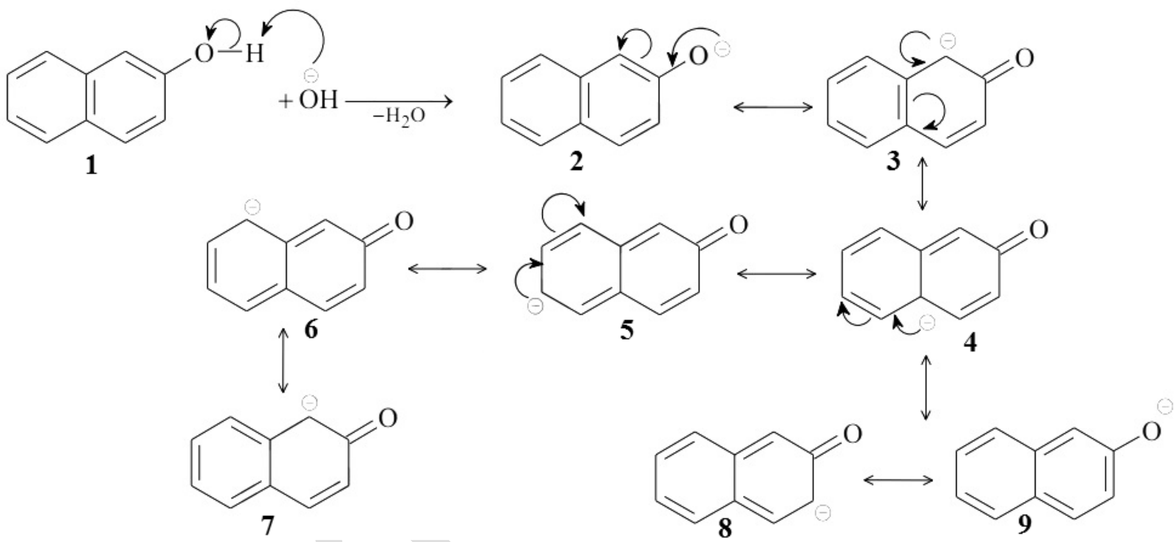
$$F = -\frac{Uo}{2}[\frac{1}{a} - \frac{1}{3}X\frac{3x^2}{a^3}]$$

21. Ans. 2



The compound M has two stereoisomers.

22. Ans. 9



23. Ans. 8

The possible structures for N2O3 are as follows:

Therefore, the total number of lone pairs in N2O3 is 8.

24. Ans. 4.18 (approximately 4)

The compounds are as follows:

$[Fe\ (SCN)\ _6]^{3-}$ and $[Fe(CN)_6]^{3-}$

The electronic configuration of Fe^{3+} in both the compounds = $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $3d^5$

The pairing will occur only in case of $[Fe(CN)_6]^{3-}$ due to CN being strong field ligand. SCN on the other hand is a weak field ligand.

1st case = $\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)} = \sqrt{35} = 5.91BM$

2nd case: $\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3} = 1.73BM$

Difference between the spin only magnetic moment = 5.91-1.73 = 4.18

25. Ans. 4

$BeCl_2$ – sp, linear
N_3^- - sp, linear
N_2O- sp, linear
NO_2^+sp, linear
O_3- sp^2, bent
SCl_2 – sp^3, bent
$ICl_2^-$$sp^3d$, linear
I_3^- - sp^3d, linear
XeF_2 - sp^3d, linear

Only 4 compounds have linear shape with the absence of d-orbital in hybridization.

26. Ans. 3

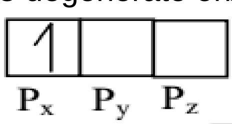
Multi electron species follows the$(n + \lambda)$ rule but Single electron species don't follow the $(n + \lambda)$ rule.

Ground state of H^- = $1s^2$

First excited state of H^- = $1s^1$, $2s^1$

Second excited state of H^- = 1s1, $2s^0$, $2p^1$

3 degenerate orbitals.



27.

$\Delta G^0 = -nFE^0 = -2X(-0.25)X96500 = 48250J/mol = 48.25$

The number of moles of M**+** oxidized using X $\longrightarrow$ Y is as follows:

$\frac{193}{48.25} = 4\text{moles}$

28.

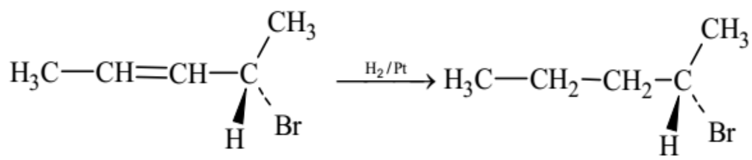
$\Delta T_f = iK_f m$

$0.0558 = iX1.86X0.01$

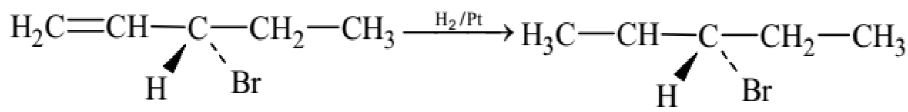
$i = 3$

Therefore the complex is $[Co(NH_3)_5Cl]Cl_2$,.

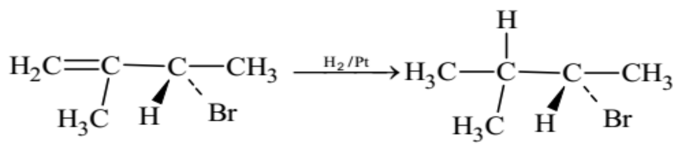
29. Ans. B, D



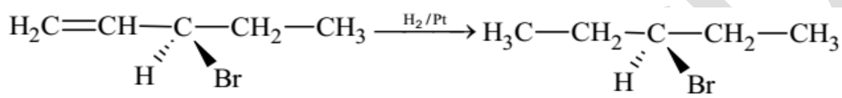
1. Optically active



2. Optically inactive

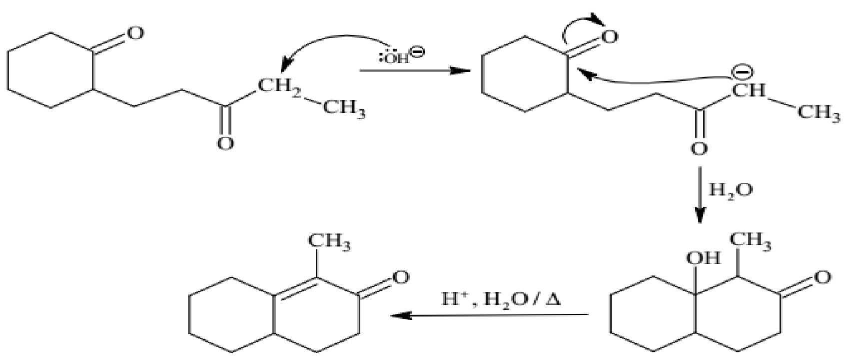


3. Optically active

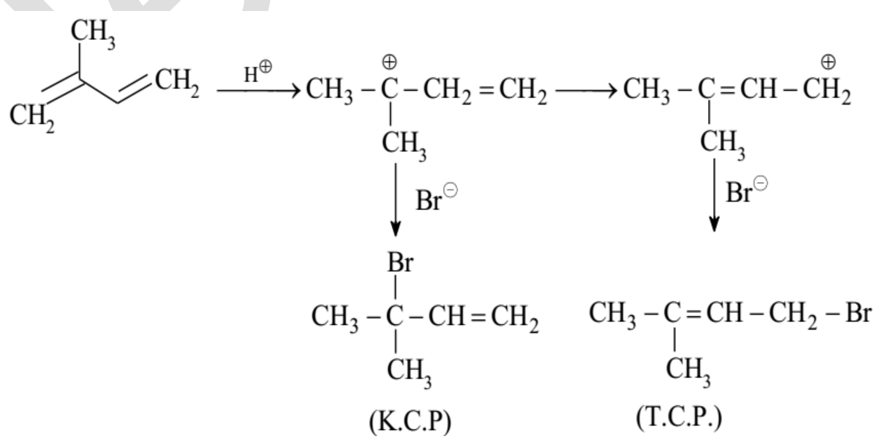


4. Optically inactive

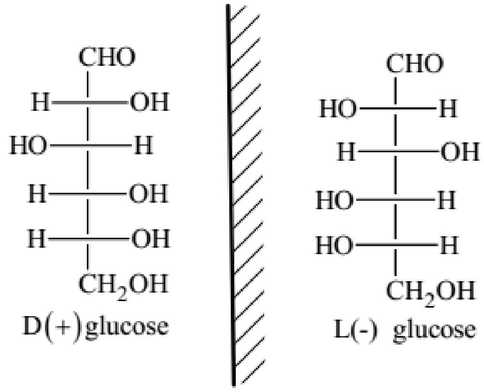
30. Ans. A.



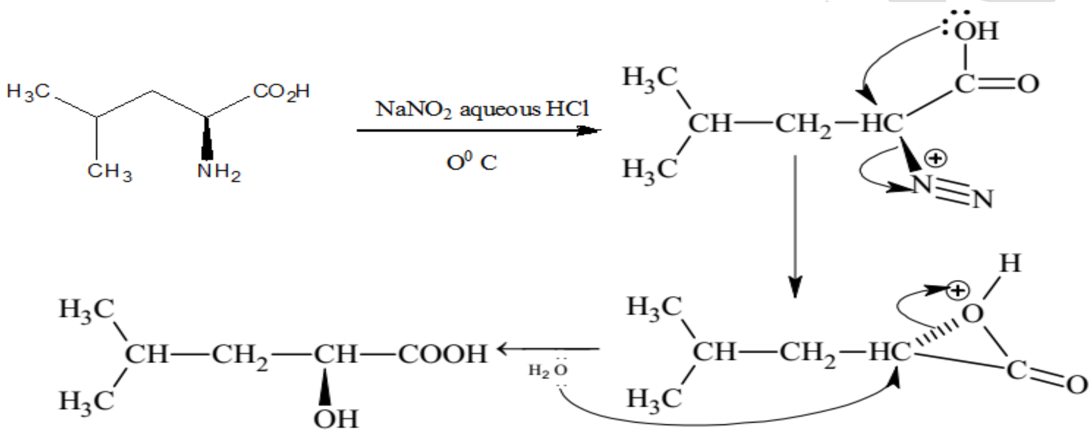
31. Ans. D.



32. Ans. A.



33. Ans. C.



34. Ans. A, B, C

Cr2+ is a reducing agent due to more stability of Cr3+.
Mn3+ is an oxidizing agent due to more stability of Mn2+.
Cr+2 = $[Ar]^{18}\ 3d^4$
Mn+3 = $[Ar]^{18}\ 3d^4$
Cr+2 $\rightarrow$ Cr^{+3} i.e. $[Ar]^{18}\ 3d^3$

Cr2+ and Mn3+ exhibit d4 electronic configuration.
Therefore, statement (D) stands out to be false.
Chromium (II) compounds are reducing agents as they are readily converted into chromium (III) compounds due to the presence of oxygen in air.

35. Ans. B, C, D

Option a is correct because while purifying Copper by electrolytic refining of blister copper Impure Cu strip is used as anode.

36. Ans. A, B

H2O2 oxidizes acidic ferrous sulphate to ferric sulphates. The reactions are as follows:

$$2Fe^{+3} + H_2O_2 + 2OH^- \longrightarrow 2Fe^{+2} + 2H_2O + O_2$$

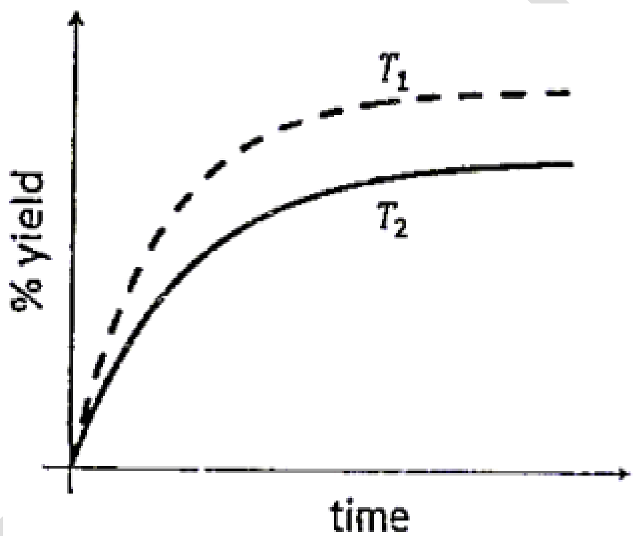
$$Na_2O_2 + H_2O \longrightarrow H_2O_2 + NaOH$$

37. Ans. C.

$N_2(g) + 3H_2(g)$

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) ; \Delta H < 0$$

In case of exothermic reactions high temperature favours the reaction in backward direction. Therefore, the yield of reaction decreases with increasing the temperature. Therefore the graph would be as follows:



38. Ans. A.

Number of oxygen atoms per unit cell in ccp = 4 (O^{-2})
Number of octahedral voids per unit cell = 4 (Al^{+3})
Number of Tetrahedral voids per unit cell = 8 (Mg^{+2})
Total negative charge due to oxygen atoms = 8
Net charge must be zero.
m4(3) + 2n(8) + 4(–2) = 0
3m + 4n = 2

a. $\frac{3}{2} X \frac{4}{8} = 2$ (correct) therefore, $m = \frac{1}{2}; n = \frac{1}{8}$

b. $3X1 + 4X\frac{1}{4} = 4$ (incorrect as $4 \neq 2$)

c. $3X\frac{1}{2}+4X\frac{1}{2}=\frac{7}{2}$ (incorrect as 7/2 ≠ 2)

d. $3X\frac{1}{4}+4X\frac{1}{8}=\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$ (incorrect as 5/4 ≠ 2)

39. Ans.

Siderite $FeCO_3$
Malachite $CuCO_3.Cu(OH)_2$
Bauxite $AlOx(OH)_{3-2x}$
Calamine $ZnCO_3$
Argentite Ag_2S
Therefore, (A) → (P), (Q), (S) ; (B) → (T) ; (C) → (Q) , (R) ; (D) → (R)

40. Ans.

As $H_2O(\ell) \rightleftharpoons H_2O(s)$
(A) → (R), (T)
$\Delta G = 0$ and $\Delta U = 0$, $\Delta S_{sys} < 0$
Therefore, (B) → (P), (Q), (S)
In case of free expansion w = 0, $\Delta U = 0$, q = 0
(C) → (P), (Q), (S)
q = 0, $\Delta U = 0$, w = 0
(D) → (S), (T)
Therefore, we get that (A) → (R), (T); (B) → (P), (Q), (S); (C) → (P), (Q), (S); (D) → (S), (T).

41. $\frac{5}{4}cos^2 2x + cos^4 x + sin^4 x + cos^6 x + sin^6 x = 2$

$\frac{5}{4}(2cos^2 x - 1)^2 + (cos^2 x + sin^2 x)^2 - 2sin^2 x cos^2 x + (cos^2 x + sin^2 x)^3 - 3sin^2 x cos^2 x(cos^2 x + sin^2 x) = 2$

$\frac{5}{4}(4cos^4 x + 1 - 4cos^2 x) + 1 - 2sin^2 x cos^2 x + 1 - 3sin^2 x cos^2 x = 2$

$5cos^4 x + \frac{5}{4} - 5cos^2 x + 2 - 5sin^2 x cos^2 x = 2$

$5cos^4 x + \frac{5}{4} - 5cos^2 x - 5sin^2 x cos^2 x = 0$

$4cos^4 x + 1 - 4cos^2 x - 4sin^2 x cos^2 x = 0$

$4cos^2 x(cos^2 x - 1) + 1 - 4sin^2 x cos^2 x = 0$

$-4sin^2 x cos^2 x + 1 - 4sin^2 x cos^2 x = 0$

$8sin^2 x cos^2 x = 1$

$2sin^2 2x = 1$

$2 - 2cos^2 2x = 1$

1 = 1 + 4cos4x

So, cos4x = 0

So, 4x = (2n+1)$\frac{\pi}{2}$

So, x = (2n+1)$\frac{\pi}{8}$ $\forall x \epsilon [0, 2\pi]$

So, x = $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots \ldots \ldots \ldots \ldots .. \frac{15\pi}{8}$

So, 8 values of x can be possible.

42.

The image of the vertex (0,0) of the parabola y^2 = 4x about the line x + y + 4 = 0 is

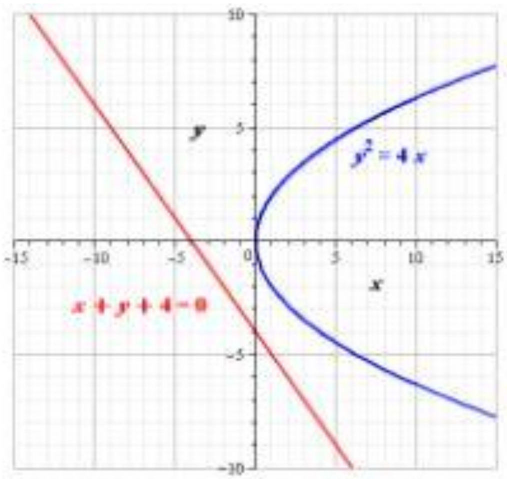
$$\frac{x-0}{1} = \frac{y-0}{1} = -2\frac{(0+0+4)}{2}$$

So, x = -4 , y = -4

The image of the focus (1,0) of the parabola y^2 = 4x about the line x + y + 4 = 0 is

$$\frac{x-1}{1} = \frac{y-0}{1} = -2\frac{(1+0+4)}{2}$$

So, x = -4 and y = -5



So, the equation of the mirror image parabola is $(x+4)^2$ = - 4(y+4)

And y = - 5 intersects it.

So, $(x+4)^2$ = - 4(-5+4) = 4

So, x + 4 = ± 2

So, x = -6 , -2

So, A is (-6 , -5) and B is (-2 , -5)

Distance between A and B is 4.

43.

Let n be no. of times the coin be tossed.

Probability of atleast two heads = 0.96

P(head) = 1/2 , p(tail) = 1/2

probability of atleast 2 head = 1 – P(no head) – P(1 head)

By using binomial distribution , we get

probability of atleast 2 head = 1 – $^nC_0(1/2)^n$ - $^nC_1 \times (1/2) \times (1/2)^{n-1}$

$0.96 = 1 - (1/2)^n - n\,(1/2)^n$

$(n + 1)/2^n = 0.04$

By hit and trial , we get n = 8.

44.

Total no. of boys = 5 and girls = 5

n = no. of ways in which 5 boys and the 5 girls stand in a queue such that all the girls stand consecutively.

So, n = 6 × 5! × 5! = 6! × 5!

And m = no. of ways in which 5 boys and the 5 girls stand in a queue such that exactly the 4 girls stand consecutively in a queue.

So, m = 5 ×7! × 4! – 5 × 4! × 2 × 6! = 5 ×7! × – 5! × 6! × 2 = 5! × 6! (7 – 2)

So, m/n = [5! × 6! (7 – 2)] / (6! × 5!) = 7 – 2 = 5

45.

Given : Parabola , $y^2 = 4x$

So, a = 1

Since , coordinates of parabola = (a , 2a) and (a , -2a) = (1, 2) and (1 , -2)

and $2y\frac{dy}{dx} = 4$

so, $\frac{dy}{dx} = \frac{2}{y}$. = 2/2 = 1

since, $m_t = 1$ so, $m_n = -1$

so, (y -2) = -1 (x – 1) is the equation of the normal

so, y + x = 3 is the equation of the normal passing through the (1 , 2) .

since, it is also the tangent of the circle $(x - 3)^2 + (y + 2)^2 = r^2$

Distance of the center of the circle from the tangent y + x = 3 is equal to radius.

So, $r = \left|\frac{(3-2-3)}{\sqrt{2}}\right| = \sqrt{2}$

So, $r^2 = 2$

46.

$f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$, [x] is the greatest integer function.

So, $f(x^2) = \begin{cases} [x^2], & x^2 \le 2 \\ 0, & x^2 > 2 \end{cases}$

$f(x^2) = \begin{cases} [x^2], & x \in [-\sqrt{2}, \sqrt{2}] \\ 0, & x \in (-\propto, -\sqrt{2}) \cup (\sqrt{2}, \propto) \end{cases}$

In -1 < x < 2

$f(x^2) = \begin{cases} 0, & x \in [-1,1] \cup (\sqrt{2}, 2) \\ 1, & x \in [1, \sqrt{2}] \end{cases}$

now, $f(x+1) = \begin{cases} [x+1], & x+1 \le 2 \\ 0, & x+1 > 2 \end{cases} = \begin{cases} [x]+1, & x \le 1 \\ 0, & x > 1 \end{cases}$

for -1< x <2

$f(x+1) = \begin{cases} 0, -1 < x < 0 \\ 1, \ 0 \le x < 1 \\ 0, x \ge 1 \end{cases}$

so, $\int_{-1}^{2} \frac{xf(x^2)}{2+f(x+1)} dx = \int_{-1}^{0} \frac{x\times 0}{2+0} dx + \int_{0}^{1} \frac{x\times 0}{2+1} dx + \int_{1}^{\sqrt{2}} \frac{x\times 1}{2+0} dx + \int_{\sqrt{2}}^{2} \frac{x\times 0}{2+0} dx$

$= 0 + 0 + \frac{1}{2}\left[\frac{2}{2} - \frac{1}{2}\right] + 0 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Hence, $4I-1 = 4\times\frac{1}{4} - 1 = 0$

47.

Inner volume $V = \pi r^2 h$

Outer volume of cylinder $V' = \pi(r+2)^2 h = \pi(r+2)^2 \times \frac{V}{\pi r^2}$

Volume of circular disc $= 2\pi(r+2)^2$

Volume of solid $= \pi(r+2)^2 h - \pi r^2 h + 2\pi(r+2)^2$

$= \pi r^2 h + 4\pi rh + 4\pi h - \pi r^2 h + 2\pi(r+2)^2$

$= 4\pi rh + 4\pi h + 2\pi(r+2)^2$

$= 4\pi r\times \frac{V}{\pi r^2} + 4\pi\times \frac{V}{\pi r^2} + 2\pi(r+2)^2$

Volume of solid, $V_s = 4v/r + 4v/r^2 + 2\pi(r+2)^2$

$$\frac{dv_s}{dr} = \frac{-4v}{r^2} - \frac{8v}{r^3} + 48\pi$$

$$\frac{dv_s}{dr} = \frac{-4v}{100} - \frac{8v}{1000} + 48\pi$$

$$0 = -\frac{48v}{1000} + 48\pi$$

So, $V = 1000\pi$

So, $V/250\pi = 4$

48.

$f(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\, dt \;\; \forall x \in R,\; f: \left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$

So, by Leibnitz formula :

$f'(x) = 2\cos^2(x^2 + \pi/6) \times 2x - 2\cos^2 x$

so, $f'(\alpha) = 4\alpha \cos^2(\alpha^2 + \pi/6) - 2\cos^2\alpha$

since, $\int_0^{\alpha} f(x)dx = f'(\alpha) + 2$

so, $\int_0^{\alpha} f(x)dx = 4\alpha \cos^2(\alpha^2 + \pi/6) - 2\cos^2\alpha + 2$

On differentiating both sides, we get

$f(\alpha) = 4\cos^2(\alpha^2 + \pi/6) + 4\alpha \times 2\cos(\alpha^2 + \pi/6) \times (-\sin(\alpha^2 + \pi/6)) \times 2\alpha - 4\cos\alpha \times (-\sin\alpha)$

So, $f(0) = 4\cos^2(\pi/6) + 0 + 0 = 4 \times (\sqrt{3}/2)^2 = 4 \times 3/4 = 3$.

49.

X ,Y are skew symmetric matrix so, $X^t = -X$ and $Y^t = -Y$

And Z is symmetric matrix so, $Z^t = Z$

(A) $(Y^3Z^4 - Z^4Y^3)^t = [(Z^4)^t(Y^3)^t - (Y^3)^t(Z^4)^t] = [-Z^4Y^3 + Y^3Z^4] = (Y^3Z^4 - Z^4Y^3)$

So, $Y^3Z^4 - Z^4Y^3$ is symmetric matrix.

(B) $(X^{44} + Y^{44})^t = (X^{44} + Y^{44})$

So, $(X^{44} + Y^{44})$ is symmetric.

(C) $(X^4Z^3 - Z^3X^4)^t = [(Z^3)^t(X^4)^t - (X^4)^t(Z^3)^t] = [Z^3X^4 - X^4Z^3] = -(X^4Z^3 - Z^3X^4)$

So, $(X^4Z^3 - Z^3X^4)$ is skew symmetric.

(D) $(X^{23} + Y^{23})^t = -X^{23} - Y^{23}$

So, $X^{23} + Y^{23}$ is skew symmetric.

(C) and (D) are correct

50.

$$\begin{bmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\(2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\(3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{bmatrix} = -648\alpha$$

$$\begin{bmatrix}\alpha^2+2\alpha+1 & 4\alpha^2+4\alpha+1 & 9\alpha^2+6\alpha+1\\\alpha^2+4\alpha+4 & 4\alpha^2+8\alpha+4 & 9\alpha^2+12\alpha+4\\\alpha^2+6\alpha+9 & 4\alpha^2+12\alpha+9 & 9\alpha^2+18\alpha+9\end{bmatrix} = -648\alpha$$

$R_2 \rightarrow R_2 - R_1$ $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix}\alpha^2+2\alpha+1 & 4\alpha^2+4\alpha+1 & 9\alpha^2+6\alpha+1\\2\alpha+3 & 4\alpha+3 & 6\alpha+3\\4\alpha+8 & 8\alpha+8 & 12\alpha+8\end{bmatrix} = -648\alpha$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix}\alpha^2+2\alpha+1 & 4\alpha^2+4\alpha+1 & 9\alpha^2+6\alpha+1\\2\alpha+3 & 4\alpha+3 & 6\alpha+3\\2 & 2 & 2\end{bmatrix} = -648\alpha$$

$C_2 \rightarrow C_2 - C_1$ $C_3 \rightarrow C_3 - C_1$

$$\begin{bmatrix}\alpha^2+2\alpha+1 & 3\alpha^2+2\alpha & 8\alpha^2+4\alpha\\2\alpha+3 & 2\alpha & 4\alpha\\2 & 0 & 0\end{bmatrix} = -648\alpha$$

$C_3 \rightarrow C_3 - 2C_2$

$$\begin{bmatrix}\alpha^2+2\alpha+1 & 3\alpha^2+2\alpha & 2\alpha^2\\2\alpha+3 & 2\alpha & 0\\2 & 0 & 0\end{bmatrix} = -648\alpha$$

$(2\alpha^2)(-4\alpha) = -648\alpha$

$-8\alpha^3 = -648\alpha$

So, $\alpha = 0$ and $\alpha^2 = 81$

So, $\alpha = 0, 9, -9$

(B) and (C) are correct

51.

Since P_3 is the plane passing through the intersection of P_1 and P_2.

So. equation of P_3 is $P_2 + \alpha P_1 = 0$

$(x + z - 1) + \alpha y = 0$ is the equation of P_3.

Since, distance of P_3 from (0 , 1 , 0) is 1.

So, $\left|\frac{(\alpha-1)}{\sqrt{1+1+\alpha^2}}\right| = 1$

$\alpha^2 - 2\alpha + 1 = \alpha^2 + 2$

so, $\alpha = -1/2$

so, $(x + z - y/2 - 1) = 0$ is the equation of P_3.

$(2x + 2z - y - 2) = 0$ is the equation of P_3.

Distance of P_3 from (α, β, γ) is 2

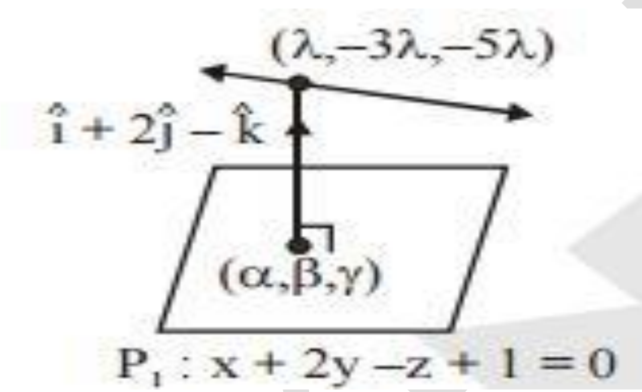
So, $\left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{9}}\right| = 2.$

So, $2\alpha - \beta + 2\gamma - 2 = \pm 6$

So, $2\alpha - \beta + 2\gamma = 8 , -4$

(B) and (D) are correct

52.



It is given that all the points of straight line 'L' equidistant from the plane $P_1 : (x+ 2y - z +1 = 0)$ and $P_2 : (2x - y +z - 1 = 0)$

So, L is parallel to the line of intersection of the plane P_1 and P_2 .

Let $\hat{n}$ be the vector along L.

So, $\hat{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

So, Equation of line L is

$\frac{x-0}{1} = \frac{y-0}{-3} = \frac{z-0}{-5} = \delta$

So, x = , y = - 3 δ and z = - 5 δ

$(\alpha , \beta , \gamma)$ be the coordinate of the feet of perpendicular from L to the plane P_1.

$\frac{x-\delta}{1} = \frac{y+3\delta}{2} = \frac{z+5\delta}{-1} = m$

So, $x = \delta + m$, $y = 2m - 3\delta$, $z = - m - 5\delta$

Since, (x , y ,z) lie in the plane P_1.

So, $\delta + m + 2(2m - 3\delta) - (-m - 5\delta) + 1 = 0$

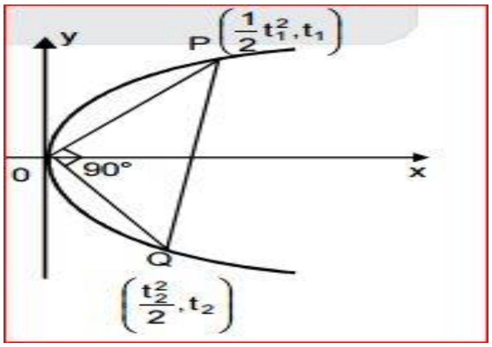
$m + 4m + m + 1 = 0$

So, m = -1/6

So, $x = -1/6 + \delta$, $y = -1/3 - 3\delta$, $z = 1/6 - 5\delta$

(A) and (B) satisfies it.

53.



Since, PQ is the diameter of the circle , so angle POQ = 90°

So, $t_1 t_2 = -4$ so , $t_2 = -4/t_1$

$OP = \sqrt{[(1/2\, t_1^2)^2 + t_1^2]}$

$OQ = \sqrt{[(1/2\, t_2^2)^2 + t_2^2]} = \sqrt{[(1/2\,(-4/t_1)^2)^2 + (-4/t_1)^2]}$

So, $OQ = \sqrt{[(64/t_1^4) + 16/t_1^2]}$

Area of $\Delta POQ = 3\sqrt{2}$ sq.units

$(1/2) \times OP \times OQ = 3\sqrt{2}$

So, $\sqrt{[(1/2\, t_1^2)^2 + t_1^2]} \times \sqrt{[(64/t_1^4) + 16/t_1^2]} = 6\sqrt{2}$

On squaring both sides , we get

$[(1/2\, t_1^2)^2 + t_1^2] \times [(64/t_1^4) + 16/t_1^2] = 72$

$16 + 16 + 4t_1^2 + 64/t_1^2 = 72$

$t_1^2 + 16/t_1^2 = 10$

$t_1^4 - 10\, t_1^2 + 16 = 0$

$(t_1^2 - 2)(t_1^2 - 8) = 0$

So, $t_1^2 = 2 , 8$

Since, P lies in the first quadrant so, $t_1 = \sqrt{2} , 2\sqrt{2}$.

(A) and (D) is the correct answer.

54.

$$(1 + e^x)\frac{dy}{dx} + ye^x = 1$$

$$\frac{dy}{dx} + \frac{ye^x}{1+e^x} = \frac{1}{1+e^x}$$

$$\frac{dy}{dx} + Py = Q$$

So, $p = \frac{e^x}{1+e^x}$

So, I.F = $e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln(1+e^x)} = 1 + e^x$

$y(1+e^x) = \int \frac{1}{1+e^x} \times (1+e^x)\mathrm{dx} = \int dx = x + c$

At x =0 , y = 2

So, 2(1 + 1) = 0 + c

So, c = 4

So, $y(1 + e^x) = x + 4$

So, $y = (x + 4) / (1 + e^x)$

(A) at x = - 4 , y = 0

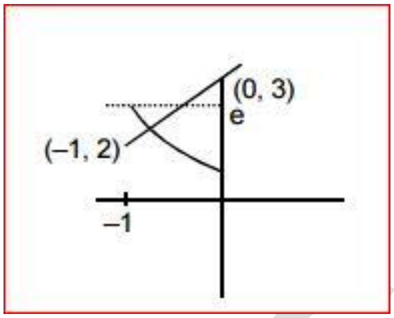
(B) at x = -2 , y ≠ 0

(C) $dy/dx = [(1+e^x) – (x + 4)e^x]/(1 + e^x)^2 = 0$

so, $1 + e^x = xe^x + 4e^x$

$1 – 3e^x = xe^x$

So, $e^x = 1/(x+3)$



So, it has a critical point between (-1 , 0)

(A) and (C) are correct.

55.

Equation of a circle is $(x – a)^2 + (y – a)^2 = r^2$ (center of the circle lies on y = x)

$x^2 + a^2 – 2ax + y^2 + a^2 – 2ay = r^2$

On differentiating this equation w.r.to x , we get

2x – 2a + 2yy' – 2ay' = 0 (y' = dy/dx)

So, a = (x + yy')/(1 + y')

On again differentiating the equation , we get

$1 + yy'' + (y')^2 – ay'' = 0$

On putting the value of a , we get

$1 + yy'' + (y')^2 - [(x + yy')/(1 + y')]y'' = 0$

$1 + y' + yy'y'' + yy'' + (y')^2 + (y')^3 – xy'' – yy'y'' = 0$

$(y – x) y'' + \{ 1 + y' + (y')^2 \}y' + 1 = 0$

On comparing it with Py'' + Qy' + 1 = 0, we get

$P = y – x$ and $Q = 1 + y' + (y')^2$

So, $P + Q = y – x + 1 + y' + (y')^2$

$P – Q = y – x – 1 – y' – (y')^2$

(B) and (C) are correct.

56.

(A) g(0) = 0, g'(0) = 0 , g'(1) ≠ 0

f(x) = $\begin{cases} -g(x) & , x < 0 \\ 0 & , x = 0 \\ g(x) & , x > 0 \end{cases}$

f'(x) = $\begin{cases} -g'(x), & x < 0 \\ g'(x), & x > 0 \end{cases}$

so, f'(0) = $\begin{cases} -g'(0) = 0, & x < 0 \\ g'(0) = 0, & x > 0 \end{cases}$

so, f is differentiable at x = 0

(B)

h(x) = $\begin{cases} e^{-x}, & x < 0 \\ e^{x}, & x > 0 \end{cases}$

h'(x) = $\begin{cases} -e^{-x}, & x < 0 \\ e^{x}, & x > 0 \end{cases}$

h'(0) = $\begin{cases} -e^{-0} = -1, & x < 0 \\ e^{0} = 1, & x > 0 \end{cases}$

so, h is not differentiable at x = 0

(C)

f(h(x)) = $\begin{cases} -g(e^{-x}) & , e^{-x} < 0, x < 0 \ (not\ possible) \\ 0 & , e^{-x} = 0, x < 0 \ (not\ possible) \\ g(e^{-x}), & e^{-x} > 0, \ x < 0 \ (possible) \end{cases}$

$\begin{cases} -g(e^{x}) & , e^{x} < 0, \ x > 0 (not\ possible) \\ 0 & , e^{x} = 0, x > 0 \ (not\ possible) \\ g(e^{x}), & e^{x} > 0, \ x > 0 \ (possible) \end{cases}$

So, f(h(x)) = $\begin{cases} g(e^{-x}), & e^{-x} > 0, \ x < 0 \\ g(e^{x}), & e^{x} > 0, \ x > 0 \end{cases}$

[f(h(x))]' = $\begin{cases} g'(e^{-x}) \times (-e^{-x}), & e^{-x} > 0, \ x < 0 \\ g'(e^{x})\, e^{x}, & e^{x} > 0, \ x > 0 \end{cases}$

[f(h(0))]' = $\begin{cases} g'^{(e^{-0})} \times (-e^{-0}) = -g'(1), & e^{-x} > 0, \ x < 0 \\ g'^{(e^{0})} e^{0} = g'(1), & e^{x} > 0, \ x > 0 \end{cases}$

So, foh is not differentiable at x = 0

(D)

f(x) = $\begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

h(x) = $e^{|x|}$

so, h(f(x))= $e^{\left|\frac{x}{|x|} g(x),\right|} = e^{|g(x)|}$

so, hof' = $e^{|g(x)|} \times |g'(x)|$

[h(f(0))]' = $e^{|g(0)|} \times |g'(0)| = e^{0}$ × g'(0) = 1 × 0 = 0

So, (A) and (D) are correct.

57.

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ *and* $g(x) = \frac{\pi}{2}\ sinx$

$f(g(x)) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$

$g(f(x)) = \frac{\pi}{2}\sin\left(\sin\left(\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)\right)$

(A)

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$, $x\epsilon R$

$sinx\ \epsilon\ [-1\ ,1]$

So, $f(x) = \sin\left(\frac{\pi}{6}\sin a\right)$, $a\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

So, $f(x) = \sin(b)$, $b\epsilon\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$

So, , $f(x)\epsilon\left[\frac{-1}{2},\frac{1}{2}\right]$

(B)

Similarly for f(g(x))

$f(g(x)) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$, g(x) $\epsilon\ \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

So, $f(g(x))\epsilon\left[\frac{-1}{2},\frac{1}{2}\right]$

(C)

$$\lim_{x\to 0}\frac{f(x)}{g(x)} = \lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x} = \lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\times\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\times\left(\frac{\pi}{2}\sin x\right)} = \frac{\pi}{6}$$

(D)

$g(f(x)) = \frac{\pi}{2}\sin\left(\sin\left(\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)\right) = 1$

Range of g(f(x)):

$f(x)\epsilon\left[\frac{-1}{2},\frac{1}{2}\right]$

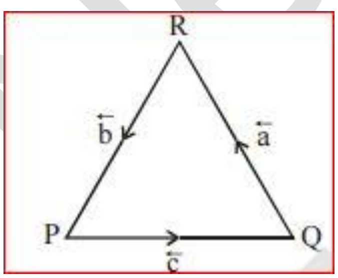
$g(f(x)) = \frac{\pi}{2}\sin(f(x)) = \frac{\pi}{2}\ \sin\left[\frac{-1}{2},\frac{1}{2}\right] = \frac{\pi}{2}\ [-0.48, 0.48] = [-0.75,0.75]$

So, Range of g(f(x)) = $[-0.75,0.75]$

So, there is no solution for $g(f(x)) = \frac{\pi}{2}\sin\left(\sin\left(\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)\right) = 1$

Hence, (A), (B) and (C) are correct.

58.



Given: PQR is a triangle in which $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$, $\vec{c} = \overrightarrow{PQ}$ such that $|\vec{a}|$=12 , $|\vec{b}| = 4\sqrt{3}$, $\vec{b}.\vec{c}$=24.

Since, by triangle law , $\vec{a} + \vec{b} + \vec{c} = 0$

So, $\vec{b} + \vec{c} = -\vec{a}$
On squaring both sides , we get
$|\vec{b}|^2 + |\vec{c}|^2 + 2\,\vec{b}.\vec{c} = |\vec{a}|^2$
$48 + |\vec{c}|^2 + 48 = 144$
So, $|\vec{c}|^2 = 48$
$|\vec{c}| = 4\sqrt{3}$
(A) $|\vec{c}|^2/2 - |\vec{a}| = 24 - 12 = 12$
(B) $|\vec{c}|^2/2 + |\vec{a}| = 24 + 12 = 36$
(D)
$\vec{a} + \vec{b} = -\vec{c}$
On squaring both sides, we get
$|\vec{b}|^2 + |\vec{a}|^2 + 2\,\vec{b}.\vec{a} = |\vec{c}|^2$
$48 + 144 + 2\,\vec{b}.\vec{a} = 48$
So, $\vec{b}.\vec{a} = -72$
(C)
$\vec{a} + \vec{b} + \vec{c} = 0$
$\vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$
So, $\vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$
So, $\vec{a} \times \vec{c} = -\vec{a} \times \vec{b}$
So $\vec{c} \times \vec{a} = \vec{a} \times \vec{b}$
$|\vec{c} \times \vec{a} + \vec{a} \times \vec{b}| = 2|\vec{a} \times \vec{b}| = 2\sqrt{[144 \times 78 - (72)^2]} = 48\sqrt{3}$.
(A) , (C) and (D) are correct

59.
(A)
$\sqrt{3} = \left|(\alpha\hat{\imath} + \beta\hat{\jmath}).\frac{(\sqrt{3}\hat{\imath}+\hat{\jmath})}{\sqrt{4}}\right|$
So, $\sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$
So. $\beta = -\sqrt{3}\alpha \pm 2\sqrt{3}$.
Also, $\alpha = 2 + \sqrt{3}\beta$ (given)
So, $\alpha = 2 + \sqrt{3}(-\sqrt{3}\alpha \pm 2\sqrt{3}) = 2 - 3\alpha \pm 6$
So, $4\alpha = 2 \pm 6 = -4 , 8$
So, $\alpha = -1 , 2$
So, $|\alpha| = 1 , 2$
(P) and (Q) are correct

(B)

$$f(x) = \begin{cases} -3ax^2 - 2 , x < 1 \\ bx + a^2 , x \geq 1 \end{cases}$$

So, $f'(x) = \begin{cases} -6ax , x < 1 \\ b,\ x \geq 1 \end{cases}$
Since, f(x) is differentiable at all x.
So, $-6a = b$……….(i)
Also, f should be continuous at all x.

So, $-3a - 2 = b + a^2$

$-3a - 2 = b + a^2 = -6a + a^2$(from (i))

$a^2 - 3a + 2 = 0$

So, a = 1, 2

(P) and (Q) are correct

(C)

ω *is a cube root of* 1 *so,* $\omega^3 = 1$

$(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0.$

Let $a = (3 - 3\omega + 2\omega^2)$

Then $a\omega = (2 + 3\omega - 3\omega^2)$ *and* $a\omega^2 = (-3 + 2\omega + 3\omega^2)$

$a^{4n+3} + (a\omega)^{4n+3} + (a\omega^2)^{4n+3} = a^{4n+3}(1 + \omega^{4n+3} + \omega^{8n+6}) = 0$

so, n shouldn't be the multiple of 3.

So, (P) , (Q) , (S) , (T) are correct.

(D)

4 = (2ab)/(a+b), 2a + 2b = ab

a , 5 , q and b are in A.P

so, 5 = a + d. , d = 5 – a.

so, b = a + 3d = a + 15 – 3a = 15 – 2a

so, b = 15 – 2a.

so, 2 (a + b) = a(15 – 2a)

$2a + 30 - 4a = 15a - 2a^2$

$2a^2 - 17a + 30 = 0$

So, a = 6 , 5/2

So, q = a + 2d = a + 10 – 2a = 10 – a

So, (q – a) = 10 – 2a = 10 – 12 , 10 – 5 = -2 , 5

So, $|q - a|$ = 2 , 5

(Q) , (T) are the correct answer.

60.

(A)

Given :$2(a^2 - b^2) = c^2$

a/Sinx = 2R

so, a = 2Rsinx

similarly, b = 2Rsiny and c = 2Rsinz

$2\times(4R^2)(\sin^2x - \sin^2y) = 4R^2\sin^2z$

$2(\sin^2x - \sin^2y) = \sin^2z$

$2\sin(x + y)\sin(x - y) = \sin^2z$

$2\sin(\pi - z)\sin(x - y) = \sin^2z$ [x + y + z = π]

$2\sin z\sin(x - y) = \sin^2z$

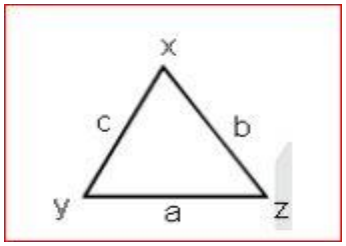
So, [sin (x – y)/sinz] = ½

So, λ = ½ (sin (x – y)/sinz = λ)

So, cos(nπ/2) = 0

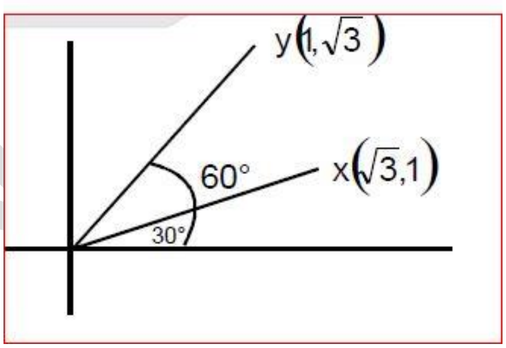
So, n = 1 , 3 , 5
(P) , (R) , (S) are the correct answers.

(B)



By Sine rule:
a/Sinx = 2R
so, a = 2Rsinx
Similarly, b = 2Rsiny and c = 2Rsinz
1 + cos2X – 2cos2Y = 2 sinX sinY
$1 + 1 – 2\sin^2 X – 2(1 – 2 \sin^2 Y) = 2 \sin X \sin Y$
$2\text{Sin}^2 Y – \sin^2 X = \sin X \sin Y$
$(1/4R^2)\ (2b^2 – a^2) = (1/4R^2)\ (ab)$
So, $2b^2 – a^2 = ab$
2b/a – a/b = 1
2/x – x =1 (a/b = x)
$x^2 + x – 2 = 0$
so, x = - 2 , 1
So, a/b = 1 (P is the only correct answer)

(C)



x(√3 , 1) , y(1 , √3) , z(β , 1 – β)
Angle between OX and OY is 30˚ and angle of its bisectors from the horizontal axis is 45˚.
So, the angle bisector of OX and OY lie along the line y = x
so, Distance of z from this line is 3/√2.

So, $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

So, 2β – 1 = ± 3

2β = 1 ± 3 = -2 , 4

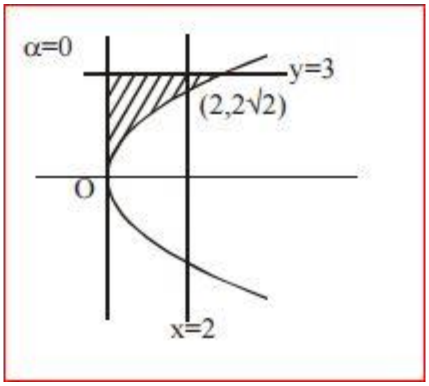
So, β = -1 , 2

So, $|\beta|$ = 1 , 2

(P) and (Q) are the correct answer.

(D)

For α = 0, F(α) is the area of region between x = 0 , x = 2, y^2 =4x and y = 1 + 2 = 3

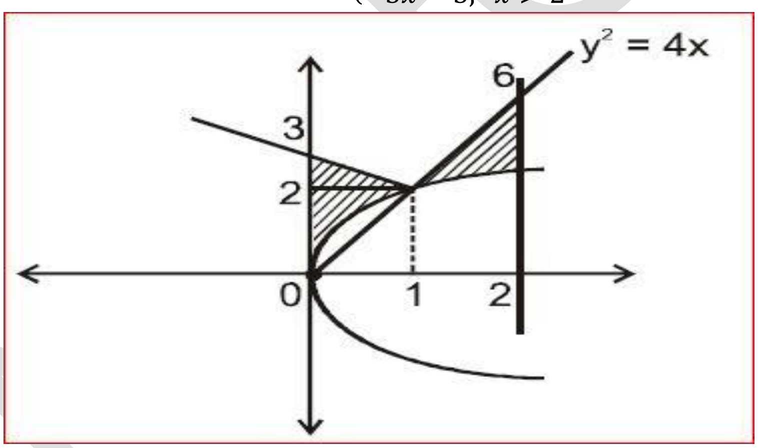


Area =F(α) = $6-\int y dx = 6-\int_0^2 2\sqrt{x}dx = 6-\frac{4}{3}\times x^{\frac{3}{2}} = 6-\frac{4}{3}\times 2\sqrt{2} = 6-\frac{8}{3}\sqrt{2}$

So, F(α) + $\frac{8}{3}\sqrt{2}$ = 6

For α = 1, F(α) is the area of region between x = 0 , x = 2, y^2 =4x and y = $|x-1|+|x-2|+x$

y = $|x-1|+|x-2|+x = \begin{cases} 3-x, & 0<x<1 \\ 1+x, & 1<x<2 \\ 3x-3, & x>2 \end{cases}$



Area of shaded region = F(α) = $\frac{1}{2}\times(2+3)\times 1+\frac{1}{2}\times(2+3)\times 1-\int_0^2 2\sqrt{x}dx = 5-\frac{8}{3}\sqrt{2}$

So, F(α) + $\frac{8}{3}\sqrt{2}$ = 5

So, (S) and (T) are the correct answers.