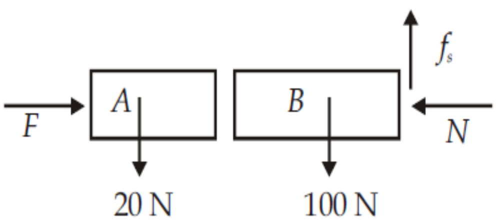
1. Ans. 3.
Since both the stones are in air (From t = 0 to t = 8 sec), so the relative distance between them Δx is
$\Delta x = x_2 - x_1 = 30t$
$\Rightarrow \Delta x \propto t$
When second stone hits the ground and first stone is in air, stone one has acceleration with respect to stone two.
Hence graph (3) is the correct description.

2. Ans. 2.
$$T^2 = \frac{4\pi^2 L}{g} \quad \Rightarrow \quad g = \frac{4\pi^2 L}{T^2}$$
$$\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2 \times \frac{\Delta T}{T} \times 100$$
$$= \frac{0.1}{20} \times 100 + 2 \times \frac{1}{90} \times 100$$
$= 0.5 + 2.2 = 2.7$ (Which is close to 3 %)

3. Ans. 3.


For block A force in equilibrium are $m_1 g = \mu_1 F$
$$20 = 0.1 \times F \quad \Rightarrow \quad F = \frac{20}{0.1} = 200\ N$$
Frictional force on block A is in upward direction $= \mu_1 F = 0.1 \times 200 = 20N$
Block A exerts frictional force of 20 N on block B in downward direction
$\therefore$ For block B, force in equilibrium is
$\mu_2 F = m_2 g + \mu_1 F = 100 + 20 = 120\ N$
So for the system to remain in equilibrium, f_s has to equal to 120N

4. Ans. 3.
Energy before collision:
$$E_1 = \frac{1}{2}m(2v)^2 + \frac{1}{2} \times 2m(v)^2 = 2mv^2 + mv^2 = 3mv^2$$

After collision:
Applying law of conservation of momentum
$3mv' = \sqrt{2} \times 2mv$ [V is their combined velocity after collision]

$$\Rightarrow v' = \frac{2\sqrt{2}}{3}v$$

Energy after collision:

$$E_2 = \frac{1}{2} \times 3m. \times \left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4v^2}{3}$$

Hence change in energy is

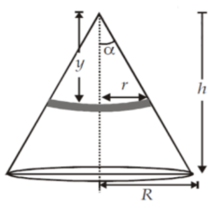
$$\Delta E = E_1 - E_2 = 3v^2 - \frac{4v^2}{3} = \frac{5v^2}{3}$$

Hence % loss in energy is

$$\frac{(E_1 - E_2)}{E_1} \times 100 = \frac{5v^2}{3} \times \frac{1}{3v^2} \times 100 = \frac{5}{9} \times 100 = 55.6\% \approx 56\%$$

5. Ans. 2.

Let y_{cm} be the position of the COM of the cone



$dm = \pi r^2 \ \rho \ dy$ [ρ is density]

Hence distance of center of mass is given by

$$y_c m = \frac{\int y \ dm}{\int dm} = \int_0^h \frac{3\pi r^2 \ y \ dy \ \rho}{\pi R^2 \ h \ \rho} = \frac{3h}{4}$$

6. Ans. 3.

Let the radius of the sphere be R and volume of sphere = $\frac{4}{3}\pi R^3$

The mass of the sphere be M. Hence density be

$$\rho = \frac{3M}{4\pi R^3}$$

Since a cube of maximum possible volume is cut. So

$2R = \sqrt{3}s$ [S is side length of the cube]

$$\Rightarrow s = \frac{2R}{\sqrt{3}}$$

Hence changed mass is

$$M' = \frac{2M}{\sqrt{3}\ \pi}$$

Moment of inertia of the cube is

$$I = \frac{M's^2}{6} = \frac{4\ MR^2}{9\sqrt{3}\ \pi}$$

7. Ans. 2.

Potential inside a solid sphere of radius R is given by:

$$V = \frac{-GM}{2R^3}\left(3R^2 - r^2\right)$$

Let V_1 be the potential at a distance r= R/2 from center due to entire sphere is

$$V_1 = \frac{-GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11\,GM}{8R}$$

Let V_2 be the potential at center of hollow sphere of radius R/2

$$V_2 = \frac{3}{8}\frac{GM}{R}$$

Hence net potential is

$$V = V_1 + V_2 = -\frac{11}{8}\frac{GM}{R} + \frac{3}{8}\frac{GM}{R} = -\frac{GM}{R}$$

8. Ans. 1.

Time period is given by

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}} \quad \text{and} \quad T_M = 2\pi\sqrt{\frac{l_2}{g}}$$

Squaring T_1 and T_2 and taking ratios of their squares we get

$$\frac{{T_M}^2}{{T_1}^2} = \frac{l_2}{l_1}$$

$$\Rightarrow \quad \frac{{T_M}^2}{{T_1}^2} - 1 = \frac{l_2}{l_1} - 1 = \frac{l_2 - l_1}{l_1}$$

Young's modulus is given by

$$Y = \frac{Mg}{A} \times \frac{l_1}{l_2 - l_1}$$

$$\therefore \quad \frac{1}{Y} = \frac{A}{Mg} \times \frac{l_2 - l_1}{l_1} = \frac{A}{Mg} \times \left[\frac{{T_M}^2}{{T_1}^2} - 1\right]$$

9. Ans. 3.

As given

$$P = \frac{U}{3V} \propto T^4$$

According to ideal gas equations PV = nR'T ------------- ideal gas equation

$$\frac{nR'T}{V} \propto T^4$$

$$\frac{1}{V} \propto T^3$$

$$\frac{3}{4\pi R^3} \propto T^3$$

$$\Rightarrow\ T \propto \frac{1}{R}$$

10. None of the given option is correct.
    Entropy(s) is a state function so depends on initial temperature (T) and final temperature (T').

$$\mathrm{ds} = \frac{\mathrm{dQ}}{\mathrm{T}} = \mathrm{ms}\frac{\mathrm{dT}}{\mathrm{T}}$$

$$\Delta s = \int \mathrm{ds} = \mathrm{ms}\int_{T}^{T'} \frac{\mathrm{dT}}{\mathrm{T}} = 1\log\frac{\mathrm{T'}}{\mathrm{T}} = \log\frac{473}{373}$$

    So no option matches.

11. Ans. 3.
    Average time taken collisions

$$t = \frac{\lambda}{v_{rms}} \qquad (\lambda = \text{mean free path})$$

$$\lambda = \frac{V}{\sqrt{2}N\pi d^2}$$

    Where, N = Number of molecules , V = Volume

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow\ t \propto \frac{V}{\sqrt{T}} \quad \longrightarrow \quad T \propto \frac{V^2}{t^2}$$

    For adiabatic process $TV^{\gamma-1} =$ constant

$$\therefore\ \ t \propto V^{\gamma+\frac{1}{2}}$$

12. Ans. 2.
    A simple pendulum performs SHM
    For simple harmonic motion, Potential energy is given by

$$U = \frac{1}{2}m\omega^2x^2$$

$$U_{max} = \frac{1}{2}m\omega^2A^2$$

    Where A= amplitude of SHM
    Kinetic energy is given by

$$E = \frac{1}{2}m\omega^2 2(A^2 - x^2)$$

    Hence graph 2 represents required plot.

13. Ans. 2.
    This is a problem on Doppler's Effect.
    As train approaches the observer the frequency of whistle heard is

$$f_1 = f\left(\frac{v}{v-v_s}\right) = 1000 \times \frac{320}{320-20} = \frac{1000\times 320}{300} = 1066.67 \text{ Hz}$$

Where v is velocity of sound in air and v_s is velocity of train and f is the frequency of whistle blown by train
As train goes away from observer the frequency of whistle heard is

$$f_2 = f\left(\frac{v}{v+v_s}\right) = 1000 \times \frac{320}{320+20} = \frac{1000\times 320}{340} = 941.17 \text{ Hz}$$

Hence % change in the frequency of the train is

$$\frac{f_1-f_2}{f}\times 100 = \frac{1066.67-941.17}{1000}\times 100 = \frac{125.5}{10} = 12.55\% \approx 12\%$$

14. Ans. 1.
The upper half of the cylinder will act as the positive end of a dipole and the lower half as the negative end and since it is given that the charge densities are same in magnitude so the magnitude of positive and negative charge will be same.

The field lines should be like that of a dipole.
So option (1) is correct.

15. Ans. 3,4.
Potential at the surface is given by

$$V_o = \frac{kQ}{R}$$

Potential inside the sphere is given by

$$V_{in} = \frac{KQ}{2R^2}(3R^2 - r^2)$$

$$V_{out} = \frac{KQ}{r}$$

Where r>>R.

$$V_C = \frac{3V_O}{2} \text{ is only possible when } R_1 = r = R. \quad (1)$$

$\frac{5}{4}V_0$ is possible inside the sphere

$$\therefore \frac{5}{4}\times\frac{kQ}{R} = \frac{KQ}{2R^2}(3R^2-r^2)$$

$$R_2 = r = \frac{R}{\sqrt{2}} \quad (2)$$

Thus we get

$\frac{3V_0}{4}$ is possible only outside the sphere

$$\therefore \frac{3}{4}\times\frac{kQ}{R} = \frac{kQ}{r}$$

$$\therefore R_3 = r = \frac{4R}{3} \quad (3)$$

$\frac{V_0}{4}$ is only possible when $R_4 = r >> R$

$\frac{kQ}{4R} = \frac{kQ}{R_4}$

$\therefore R_4 = 4R \quad (4)$

Using equations 1,2,3,4, we get option (3, 4) correct.

16. Ans. 2.

$$Q_2 = \frac{2EC}{C+3}$$

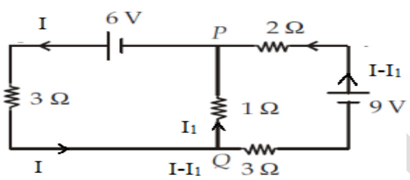
As C increases from 1 to 3 μF, Q_2 increases as E is constant. Concavity of the graph can be found out using 2nd derivative.

17. Ans. 4.

$$V = IR = \frac{I \times \rho \ell}{A}$$

$$\Rightarrow \rho = \frac{V \times A}{i\ell} = \frac{V}{v_d \cdot ne\ell} = \frac{5 \times 10^{-4}}{2.5 \times 8 \times 1.6} = \frac{2 \times 10^{-4}}{14.4}$$

$$\rho = \frac{10^{-4}}{6.4} = 1.6 \times 10^{-5} \quad \Omega - m$$



18. Ans. 3.

Here $I = I_1 + I_2$

Applying KVL,

$-6 + I_1 + 3I = 0 \quad \rightarrow \quad 3I + I_1 = 6 \qquad (1)$

$2(I - I_1) - 9 + 3(I - I_1) = 0 \quad \rightarrow \quad 5I - 6I_1 = 9 \qquad (2)$

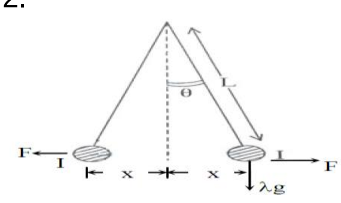
Solving (1) and (2), we get

$$I_1 = \frac{3}{23} = 0.13\ A \ \text{ from Q to P}$$

19. Ans. 1.

Net Magnetic force on both of them = 0.

20. Ans. 2.



Let T be the tension in the string

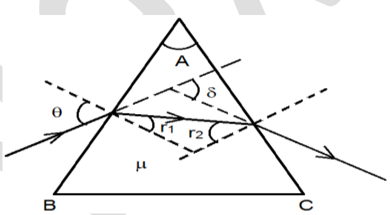
$T\cos\theta = \lambda g \quad (1)$
$T\sin\theta = \frac{\mu_0\ I^2}{4\pi\ell\sin\theta} \quad (2)$
Dividing (1) by (2) we get
$I = 2\ \sin\theta\sqrt{\pi\ell g\lambda\cos\theta}$

21. Ans. 3.
In orientation (b), M II B ❾ stable equilibrium
In orientation (d), M II (-B) ❾ unstable equilibrium

22. Ans. 4.
We know that Current in an inductor circuit is given by:
$I = I_0\ e^{-t/\tau}$
$\tau = \frac{L}{R} = \frac{1}{(5\times10^3)}$
$\therefore I = \frac{E}{R}\times e^{-t/\tau} = 0.1\times e^{\frac{10^{-3}\times 5}{10^{-3}}} = 0.1e^{-5}$
$\therefore I = \frac{0.1}{150} = 0.67\ mA$

23. Ans. 2.
$I_0 = \frac{P}{A} = \frac{P}{4\pi r^2}$
$\frac{1}{2}c\ E^2 = \frac{P}{4\pi\varepsilon_0 r^2}$

$$\Rightarrow E = \sqrt{\frac{2P}{4c\ \pi\varepsilon_0 r^2}} = \sqrt{\frac{0.1}{2\times3.14\times8.85\times10^{-12}\times3\times10^8\times1^2}} = 2.45\ \mathrm{V/m}$$



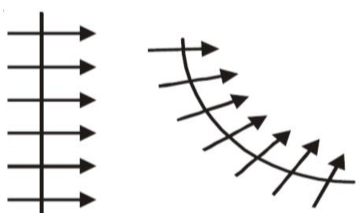
24. Ans. 1.
$r_1 + r_2 = A \quad \Rightarrow \quad r_2 = A - r_1$
At interface of prism and air, $\mu\sin r_1 = sin\theta$
For light to be transmitted $\mu\sin r_2 < 1$
$\mu\sin(A - r_1) < 1$
$\therefore (A - r_1) < \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)<\sin r_2$$
$$\therefore \sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)<\frac{\sin\theta}{\mu}$$
$$\therefore \theta>\sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

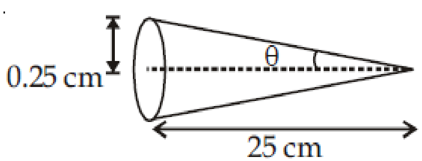
25. Ans. 4.

The light beam bends upwards because it goes from a rarer medium to denser medium.

Consider a plane wave front travelling horizontally. As it moves, its different parts move with different speeds. So, its shape will change as shown



26. Ans. 2.



$$d\sin\theta=2r\sin\theta=1.22\lambda$$
$$\because \sin\theta=\frac{x}{D}$$
$$2r\times\frac{x}{D}=1.22\lambda$$
$$x=\frac{1.22\lambda\times D}{2r}=\frac{1.22\times5\times10^{-7}\times0.25}{0.5\times10^{-2}}$$
$$\therefore x=3.05\times10^{-5}\ m=30.5\ \mu m$$

27. Ans. 1.

Total Energy = K.E. + P.E.

$$\text{Kinetic Energy }=\frac{-13.6\ Z^2}{n^2}\ eV$$
$$\text{Potential energy }=2\times\text{ Kinetic Energy }=\frac{-27.2\ Z^2}{n^2}\ eV$$

So as n decreases, KE increases and PE & TE decreases

28. Ans. 3.

Franck-Hertz experiment. – Discrete energy level.

Photoelectric effect– Particle nature of light

Davisson-Germer experiment. – Diffraction of electron beam.

29. Ans. 3.
A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz.
The frequencies of the resultant signal are therefore 2005 Hz, 2000 Hz, and 1995 Hz.

30. Ans. 1.
We know that,
For a damped pendulum
$A = A_0 e^{\frac{-bt}{2m}}$
Since m can equivalently replaced by L.
$A = A_0 e^{\frac{-Rt}{2L}}$
$\because \; L_1 > L_2$ Hence with L_1 dissipation in R in the circuit will be lower compared to L_2
So appropriate graph is (1)

31. Ans. 4.
$2C_8H_7SO_3Na + Ca^{2+} \rightarrow (C_8H_7SO_3)2Ca + 2Na^+$
Mole of Ca^{2+} exchanged by 412 g of resin = 1 mole
Mole of Ca_2+ exchanged by 1 g of resin = $\frac{1}{412}$ g.
Maximum uptake of Ca_2+ ions by the resin = $\frac{1}{412}$ moles per gram resin.

32. Ans. 1.
For BCC crystal structure
$\sqrt{3}a = 4r$
$a = 4.29\text{Å}$
$\therefore r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4} \times 4.29\text{Å} = 1.86\text{Å}$

33. Ans. 3.
$E_n = -13.6 \times \frac{Z^2}{n^2}$ eV
For hydrogen Z = 1
$E_n = -13.6 \times \frac{1}{n^2}$ eV
For excited state, n = 2, 3, 4,
∴for n = 2,
$E_2 = \frac{-13.6}{2^2} = -3.4\ eV$

34. Ans. 2.

For ion-dipole interaction

$$F \propto \mu \frac{dE}{dr} \quad \text{(where } \mu \text{ is dipole moment of dipole and r is distance between ion and dipole)}$$

$$\propto \mu \frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$\propto \frac{\mu}{r^3}$$

35. Ans. 4.

$$\Delta G^0{}_{rxn} = -RT \ln K_P$$

$$\Delta G^0{}_{rxn} = 2\Delta G^0{}_{NO_2} - 2\Delta G^0{}_{NO} - \Delta G^0{}_{O_2}$$

$$2\{\Delta G^0{}_{NO_2} - \Delta G^0{}_{NO}\} = -RT \ln K_P \{\Delta G^0{}_{O_2} = 0\}$$

$$2\{\Delta G^0{}_{NO_2} - 86600\} = -R(298)\ln(1.6\times 10^{12})$$

$$\Delta G^0{}_{NO_2} = 0.5[2\times 86600 - R(298)\ln(1.6\times 10^{12})]$$

36. Ans. 2.

P^0 = 185 torr

P_s = 183 torr

W_{Solute} = 1.2 g

W_{Solvent} = 100g

$$\frac{P^0 - P_S}{P_S} = \frac{\mathbf{W}_{\text{Solute}} \times MM_{Solvent}}{\mathbf{W}_{\text{Solent}} \times MM_{Solute}}$$

$$\frac{185-183}{183} = \frac{1.2\times 58}{100\times MM_{Solute}}$$

$$MM_{Solute} = \frac{1.2\times 58\times 183}{2\times 100} = 64 g/mol$$

37. Ans. 2.

$$\Delta G^0 = -RT \ln K$$
$$2494.2 = -8.314 \times 300 \ln K$$
$$\ln K = -1$$
$$\log_e K = -1$$
$$K = \frac{1}{e} = 0.3679$$
$$Q_C = \frac{[B][C]}{[A]^2} = \frac{2\times 0.5}{0.5^2} = 4$$

$$\therefore Q_C > K \Rightarrow \text{ Reverse direction}$$

38. Ans. 2.

$Cu^{2+} + 2e^- \rightarrow Cu$

$$\frac{n_{e^-}}{2} = \frac{n_{Cu}}{1}$$

Faraday's Charge = $n_{e^-} = 2$

$$\frac{2}{2} = \frac{W_{Cu}}{63.5}$$

$$W_{Cu} = 63.5g$$

39. Ans. 1.

According to the collision theory, collision between the particles of the reactant leads to the occurrence of reactions. Particles collide at a higher rate in case of first and second order reaction. But it is quite difficult for three or more than three molecules to collide simultaneously.

Due to this very low probability of colliding of molecules, the higher order reactions (>3) are quite rare. Lower chances of simultaneous collision between the reacting species results in lesser higher order reactions.

40. Ans. 1.

Valency factor of acetic acid = 1.

Normality = Molarity

Initial mole of acetic acid = $(0.06 \times 50) \times 10^{-3}$

Final mole of acetic acid = $(0.042 \times 50) \times 10^{-3}$

Moles of acetic acid adsorbed = $(0.06 - 0.042) \times 50 \times 10^{-3} = 0.9 \times 10^{-3}$

Mass of acetic acid adsorbed = $(0.9 \times 10^{-3} \times 60)$ g = 0.054 g or 54 mg

Mass of acetic acid adsorbed per gram = $\frac{54}{3}$ mg = 18 g.

41. Ans. 3.

The atomic number in isoelectronic series increases which in turn decreases the radius. Order of the radius is as follows:

N^{-3}	>	O^{-2}	>	F^-
1.71 A^0		1.40 A^0		1.36 A^0

42. Ans. 4.

The incorrect statement is Na_3AlF_6 serves as an electrolyte.

In Hall Heroult process fused Al_2O_3 in combination with Na_3AlF_6 and caF_2 is used as an electrolyte.

The reaction at cathode: $Al^{+3} + 3e^- \rightarrow Al$

The reaction at anode: $O^{2-} + C \rightarrow CO + 2e^-$

43. Ans. 1.

H_2O_2 acts as an oxidizing agent and reducing agent as well. In H2O2, 'O' is present in its intermediate oxidation state.

44. Ans. 2.

Order of solubility in water. $BeSO_4 > CaSO_4 > SrSO_4 > BaSO_4$. Hydration energy is inversely proportional to ionic size. Be^{+2} smaller in size hence, $BeSO_4$ has greater hydration enthalpy than its lattice enthalpy.

45. Ans. 4.

ICl is an interhalogen compound and hence are more reactive than constituent halogens due to the presence of polar bond in interhalogen compounds. Pure halogens on the other hand contains non-polar bond.

46. Ans. 2.

$TiCl_3$ ❼ Ziegler-Natta Polymerization

$PdCl_2$ ❼ Wacker process

$CuCl_2$ ❼ Deacon's process

V_2O_5 ❼ Contact process

47. Ans. 4.

The order of boiling point is as follows:

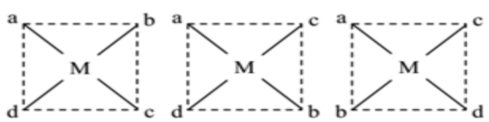
He < Ne < Kr < Xe.

The strength of intermolecular bond determines the boiling point. Increase in size increases the London forces of attraction between the molecules.

48. Ans. 2.

$[Pt (Cl) (py) (NH_3) (NH_2OH)]^+$

General form is $M_{abcd.}$



Total number of geometric isomers is 3.

49. Ans. 3.

K+ Colorless

MnO_4^- Colored

The transfer of O to Mn results in appearance of color of MnO_4^-.

Ligand ❼ Metal charge transfer transition

50. Ans. 1.

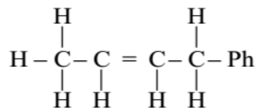
Both N_2 and O_2 are less reactive at normal conditions. They require high temperature to react.

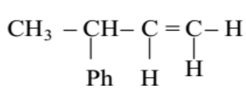
51. Ans. 1.

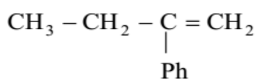
$$Br\% = \frac{80}{188} \times \frac{\text{Weight of AgBr}}{\text{Weight of O.S.}} \times 100$$

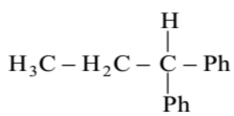
$$= \frac{80}{188} \times \frac{141}{250} \times 100 = 24\%$$

52. Ans. 1.

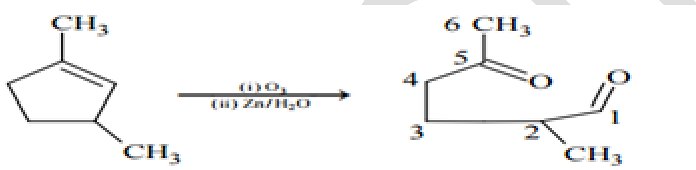
(1) 1 – Phenyl – 2- butene →  Shows Geometrical isomerism

(2) 3 – Phenyl – 1- butene →  No

(3) 2 – Phenyl – 1- butene →  No

(4) 1, 1 – Diphenyl – 1- propane →  No

53. Ans. 2.



5 - keto- 2 - methyl hexanal

Alkenes undergo oxidation during ozonolysis and forms carbonyl compounds.

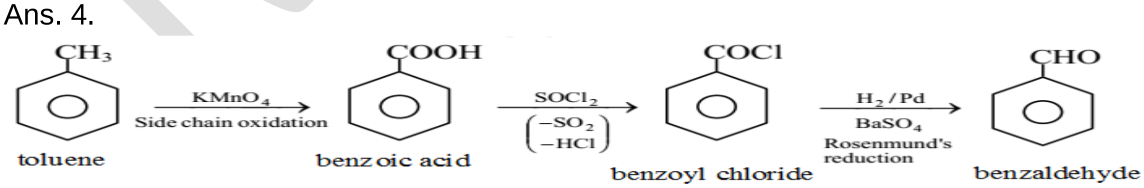
54. Ans. 4.

Swarts reaction leads to the synthesis of alkyl fluoride.

$$2R-Cl + Hg_2F_2 \xrightarrow{\Delta} 2R-F + Hg_2Cl_2$$

Alkyl halides react with heavy metal fluorides and produces alkyl fluoride.

55. Ans. 4.



56. Ans. 3.

57. Ans. 2.
Glyptal is used to manufacture paints and lacquers.

58. Ans. 1.
Vitamin C is a water soluble vitamin. It gets excreted out of the body through urine. Therefore, it is necessary to supply Vitamin C continuously to the body.

59. Ans. 3.
Except Phenelzine all others are antacid. It is used as an antidepressant.

60. Ans. 1.
Except $Zn_2\,[Fe\,(CN)_6]$ which produces Bluish white in color, all others are yellow colored compounds.

61. Ans. 1.
Total we have 8 elements implies, the number of subsets of the set $A \times B$ each having at least three elements is= ${}^8_3C + {}^8_4C + {}^8_5C + {}^8_6C + {}^8_7C + {}^8_8C = 56 + 70 + 56 + 28 + 8 + 1 = 219$.
Where ${}^n_rC = \frac{n!}{(r!(n-r)!)}$.

62. Ans. 3.
Given $\frac{z_1 - 2z_2}{2 - z_1\bar{z_2}}$ is unimodular implies

$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z_2}}\right| = 1$

$|z_1 - 2z_2| = |2 - z_1\bar{z_2}|$

Squaring on both side

$|z_1 - 2z_2|^2 = |2 - z_1\bar{z_2}|^2$

$|z_1|^2 + 4|z_2|^2 - 2z_1\bar{z_2} - 2\bar{z_1}z_2 = 4 + |z_1|^2|z_2|^2 - 2z_1\bar{z_2} - 2\bar{z_1}z_2$

$|z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$

$(|z_1|^2 - 1)(|z_2|^2 - 4) = 0$

$|z_1| = 1$ Or $|z_2| = 2$.

63. Ans. 3.
Given that α and β are roots of $x^2 - 6x - 2 = 0$ and $a_n = \alpha^n - \beta^n$

$\frac{a_{10} - 2a_8}{2a_9} = \frac{a_{10}}{2a_9} - 2\frac{a_8}{a_9}$

Find $\frac{a_{n+1}}{a_n} = \frac{\alpha_{n+1}-\beta_{n+1}}{\alpha^n-\beta^n} = \frac{\alpha^n.\alpha-\beta^n\beta}{\alpha^n-\beta^n} = \frac{\alpha^n.\alpha-\beta^n\alpha+\beta^n\alpha+\alpha^n\beta-\alpha^n\beta-\beta^n\beta}{\alpha^n-\beta^n}$

$= \frac{(\alpha+\beta)(\alpha^n-\beta^n)-\alpha\beta(\alpha^{n-1}-\beta^{n-1})}{\alpha^n-\beta^n}$

$= (\alpha+\beta)-\alpha\beta\frac{a_{n-1}}{a_n}$

$\frac{a_{n+1}}{a_n} = (\alpha+\beta)-\alpha\beta\frac{a_{n-1}}{a_n}$

For $n = 9$

$\frac{a_{10}}{a_9} = 6 + 2\frac{a_8}{a_9}$

$\frac{a_{10}}{a_9} - 2\frac{a_8}{a_9} = 6$

$\frac{a_{10}-2a_8}{a_9} = 6$

$\frac{a_{10}-2a_8}{2a_9} = 3.$

64. Ans. 4.

Given $= \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$, $A^T = \begin{bmatrix}1 & 2 & a\\ 2 & 1 & 2\\ 2 & -2 & b\end{bmatrix}$ and $I = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$AA^T = \begin{bmatrix}1+4+4 & 2+2-4 & a+4+2b\\ 2+2-4 & 4+1+4 & 2a+2-2b\\ a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}$

Given $AA^T = 9I$

$$\begin{bmatrix}9 & 0 & a+4+2b\\ 0 & 9 & 2a+2-2b\\ a+4+2b & 2a+2-2b & 9\end{bmatrix} = \begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

We have two equations $a+2b=-4$ and $a-b=-1$

$(a,b) = (-2,-1)$.

65. Ans. 3.

Given equations

$(2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 + x_2(-3-\lambda) + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$Det = 0$

$$\begin{vmatrix}2-\lambda & -2 & 1\\ 2 & -3-\lambda & 2\\ -1 & 2 & -\lambda\end{vmatrix} = 0.$$

$C_1 \to C_1 - C_3$

$$\begin{vmatrix} 1-\lambda & -2 & 1 \\ 0 & -3-\lambda & 2 \\ -1+\lambda & 2 & -\lambda \end{vmatrix} = 0.$$

$\rightarrow R_1 + R_3$

$$\begin{vmatrix} 0 & 0 & 1-\lambda \\ 0 & -3-\lambda & 2 \\ -1+\lambda & 2 & -\lambda \end{vmatrix} = 0.$$

$(1-\lambda)(-1+\lambda)(\lambda+3) = 0$

$\lambda = 1, -3, 1.$

66. Ans. 2.

We required to find the number of integers greater than $6,000$ that can be formed, using the digits $3,5,6,7$ and 8 without repetition:

First digit 6 is fixed so

6 __ __ __ __ implies $4 \times 3 \times 2 = 24$

7 __ __ __ __ implies $4 \times 3 \times 2 = 24$

8 __ __ __ __ implies $4 \times 3 \times 2 = 24$

72 Four digit numbers are greater than 6000

$5! = 120$ Five digit numbers are greater than 6000

Required is $= 120 + 72 = 192.$

67. Ans. 1.

Consider the sum $\left(1 - 2\sqrt{x}\right)^{50} + \left(1 + 2\sqrt{x}\right)^{50}$

$\left(2\sqrt{x} - 1\right)^{50} + \left(2\sqrt{x} + 1\right)^{50} = 2\{{}^{50}_{0}C\left(2\sqrt{x}\right)^{50} + {}^{50}_{2}C\left(2\sqrt{x}\right)^{48} + \cdots\}$

Substitute $x = 1$ we get,

$(2-1)^{50} + (2+1)^{50} = 2\{{}^{50}_{0}C(2)^{50} + {}^{50}_{2}C(2)^{48} + \cdots\}$

$1 + 3^{50} = 2(sum)$

68. $sum = \frac{1+3^{50}}{2}$Ans. 2.

Given m is the $A.M.$ of two distinct real numbers l and n

Implies $m = \frac{l+n}{2}$.

$\frac{l+n}{2}, l, G_1, G_2, G_3, n$ are in $G.P$

$n = lr^4$

$G_1^4 + 2G_2^4 + G_3^4 = (lr^4)^4 + 2(lr^4)^4 + (lr^3)^4$

$= l^4 r^4 [1 + 2r^4 + r^8]$

$= l^4 \times \frac{n}{l}[1 + 2 \times \frac{n}{l} + \frac{n^2}{l^2}]$

$= ln(l+n)^2 = ln(2m)^2 = 4lnm^2.$

69. Ans. 2.

Given series is

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots.$

n^{th} term is $=\frac{1^3+2^3+3^3+\cdots+n^3.}{1+3+5+\cdots.+(2n-1)}=\frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n}{2}[2n]}=\frac{n^2+2n+1}{4}$

Sum of the n terms is $=\frac{1}{4}\left[\sum n^2+2\sum n+\sum 1\right]$

$n=9$

$$=\frac{1}{4}\left[\frac{9\times(9+1)\times(18+1)}{6}+2\frac{9\times 10}{2}+9\right]$$

$$=\frac{1}{4}[285+90+9]=\frac{1}{4}[384]=96$$

70. Ans. 3.

$$\lim_{x\to 0}\frac{(1-cos\ 2x)(3+cos\ x)}{x\ tan\ 4x}$$

$$\lim_{x\to 0}\frac{2sin^2x(3+cos\ x)}{x\ tan\ 4x}$$

$$\lim_{x\to 0}\frac{2xsin^2x(3+cos\ x)}{x^2\ tan\ 4x}$$

$$\lim_{x\to 0}2\left(\frac{sin^2x}{x^2}\right)\left(\frac{3+cos\ x}{\left(\frac{tan\ 4x}{x}\right)}\right)$$

$$\lim_{x\to 0}2\left(\frac{sin^2x}{x^2}\right)\left(\frac{3+cos\ x}{\left(\frac{tan\ 4x}{4x}\right)\times 4}\right)$$

$$=\frac{2\times 1^2\times(3+1)}{1\times 4}=2$$

71. Ans. 1.

Given, $g(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2\ , & 3<x\le 5\end{cases}$

And it is differentiable

$$g'(x)=\begin{cases}\frac{k}{2\sqrt{x+1}}, & 0\le x\le 3\\ m\ , & 3<x\le 5\end{cases}$$

The limiting value of $g'(x)$ at $x=3$ should be equal.

$$\frac{k}{2\sqrt{3+1}} = m$$
$k = 4m \ldots(1)$
As $g(x)$ is differentiable it will be continuous, the limiting value of $g(x)$ at $x = 3$ should be equal

$k\sqrt{3+1} = 3m + 2$
$2k = 3m + 2 \ldots(2)$
From (1) and (2) $8m = 3m + 2$
$5m = 2$
$m = \frac{2}{5} = 0.4$
Then $k = 1.6$
Implies $m + k = 1.6 + 0.4 = 2.$

72. Ans. 4.
Given curve is $x^2 + 2xy – 3y^2 = 0$

You can rearrange the equation into two squares as follows:
$x^2 + 2xy - 3y^2 = 0$
$x^2 + 2xy + y^2 = 4y^2$
$(x + y)^2 = (2y)^2$
$x + y = \pm 2y$
$x = y \; or \; -3y$
$y = x \; or \; -\frac{x}{3}$

The point $(1,1)$ lies on one of these lines, the line $y = x$ and slope of the line is$=\ 1$. We need the slope of the normal, which is must be-1. Therefore the normal at $(1,1)$ has the equation

$y - 1 = -1\,(x - 1) = 1 - x$
$y = 2 - x$
The normal clearly does not intersect the line $y = x$ at any other point. It would however intersect the line y = -x/3. Find the intersection point by solving

$y = 2 - x \; and \; y = -\frac{x}{3}$
$-\frac{x}{3} = 2 - x$
$-x = 6 - 3x$
$2x = 6$
$x = 3$ and $y = -1$
the normal at $(1,1)$ also intersects the curve at $(3, -1)$ which is in fourth quadrant.

73. Ans. 3.

Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$$\lim_{x\to 0}\left(1+\frac{f(x)}{x^2}\right) = 3$$

$$\lim_{x\to 0}\left(1+\frac{ax^4+bx^3+cx^2+dx+e}{x^2}\right) = 3$$

$$\lim_{x\to 0}\left(ax^2+bx+c+1+\frac{d}{x}+\frac{e}{x^2}\right) = 3$$

right side it is finite value so d and e should be zero then $f(x) = ax^4 + bx^3 + cx^2$

After applying $c + 1 = 3$ implies $c = 2$

Extreme values are at $x = 1$ and $x = 2$

$f(x) = ax^4 + bx^3 + 2x^2$

$f'(x) = 4ax^3 + 3bx^2 + 4x$

$f'(1) = 4a + 3b + 4 = 0.$

$f'(2) = 32a + 12b + 8 = 0.$

Solving above two equations we get,

$a = 0.5$ and $b = -2.$

74. Ans. 4.

Let $A = \int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$

$$A = \int \frac{dx}{x^{3+2}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$$

Let $1 + \frac{1}{x^4} = y$

$-\frac{4}{x^5}dx = dy$

Substitute in equation

$$A = -\frac{1}{4}\int y^{-\frac{3}{4}}dy$$

$$A = -\frac{1}{4}\frac{y^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c$$

$$A = -y^{\frac{1}{4}} + c$$

$$A = -\left(\frac{x^4+1}{x^4}\right) + c$$

75. Ans. 3.

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$$

$I = \int_2^4 \frac{log\, x^2}{log\, x^2 + log\,(6-x)^2} dx$ ….. (1)

We have one relation $\int_a^b f(x) = \int_a^b f(a+b-x)$

$I = \int_2^4 \frac{log\,(6-x)^2}{log\,(6-x)^2 + log\,(x)^2} dx$ …. (2)

Add (1) and (2)

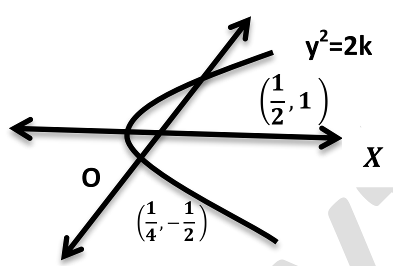
$$2I = \int_2^4 \frac{log\, x^2 + log\,(6-x)^2}{log\,(6-x)^2 + log\,(x)^2} dx$$

$2I = (4-2)$

$I = 1.$

76. Ans. 4.

$\{(x,y): y^2 \le 2x \; and \; y \ge 4x - 1\}$



Two equations are $y^2 = 2x$ and $y = 4x - 1$

Find the point of intersection

$y^2 = 2x$

$y^2 = 2\left(\frac{y+1}{4}\right)$

$2y^2 - y - 1 = 0$

$(y-1)(2y+1) = 0$

$y = 1, y = -\frac{1}{2}.$

Area= $\int_{\frac{1}{2}}^{1}\left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy = \frac{1}{4}\int_{\frac{1}{2}}^{1}(y + 1 - 2y^2) dy = \frac{1}{4}\left[\frac{y^2}{2} + y - \frac{2y^3}{3}\right]_{\frac{1}{2}}^{1} = \frac{1}{4}\left[\frac{1}{5} + 1 - \frac{2}{3} - \frac{1}{8} + \frac{1}{2} - \frac{1}{12}\right] = \frac{9}{32}.$

77. Ans. 3.

Given differential equation is

$$\frac{dy}{dx}+\frac{1}{x\log x}y=2$$

Integrating factor for the equation is $e^{\int\left(\frac{1}{x\log x}\right)dx}=\log x$

$$y\log x=c+\int 2\log x\,dx$$

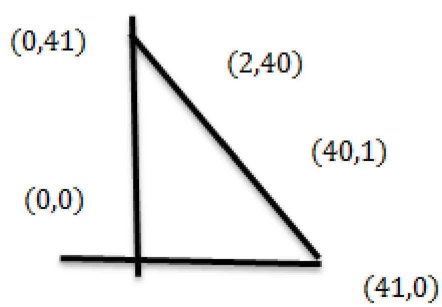
$$y\log x=c+2\{x\log x-\int x\,dx\}$$

$$y\log x=c+2x\log x-2x$$

Substitute $x=e$ then $y(e)\log e+c+2e\log e-2ey(e)=c$

78. Ans. 4.

Required number of points (From diagram) $=1+2+3+\cdots+39=\frac{39}{2}(39+1)=39\times 20=780$.



79. Ans. 3.

The given family of lines pass through the intersection of $2x-3y+4=0$ and $2x-4y+6=0$

Solving we get $x=1, y=2$.

$$m_{PM}\times m_{AB}=-1$$

$$\frac{\left[\frac{b+3}{2}-2\right]}{\left[\frac{a+2}{2}-1\right]}\times\frac{b-3}{a-2}=-1$$

$$\left[\frac{b-1}{a}\right]\left[\frac{b-3}{b-2}\right]=-1$$

$$b^2-4b+3=-a^2+2a$$

$$a^2+b^2-2a-4b+3=0$$

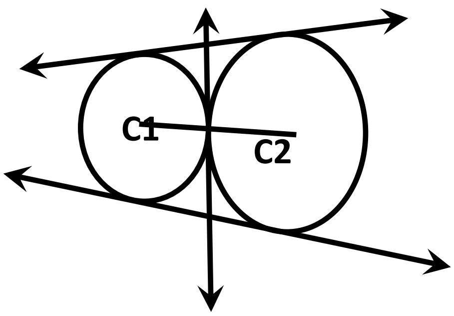
Radius $=\sqrt{1+4-3}=\sqrt{2}$

80. Ans. 3.

Given equations are

$x^2 + y^2 - 4x - 6y - 12 = 0$

$x^2 + y^2 + 6x + 18y + 26 = 0$



Rewrite them

$(x-2)^2 + (y-3)^2 = 12 + 4 + 9 = 25$

$(x+3)^2 + (y+9)^2 = -26 + 9 + 91 = 64$

Radius of the first circle is $R_1 = 5$

Radius of the second circle is $R_1 = 8$

$C_1C_2 = \sqrt{(2+3)^2 + (-9-3)^2} = \sqrt{25+144} = \sqrt{169} = 13$

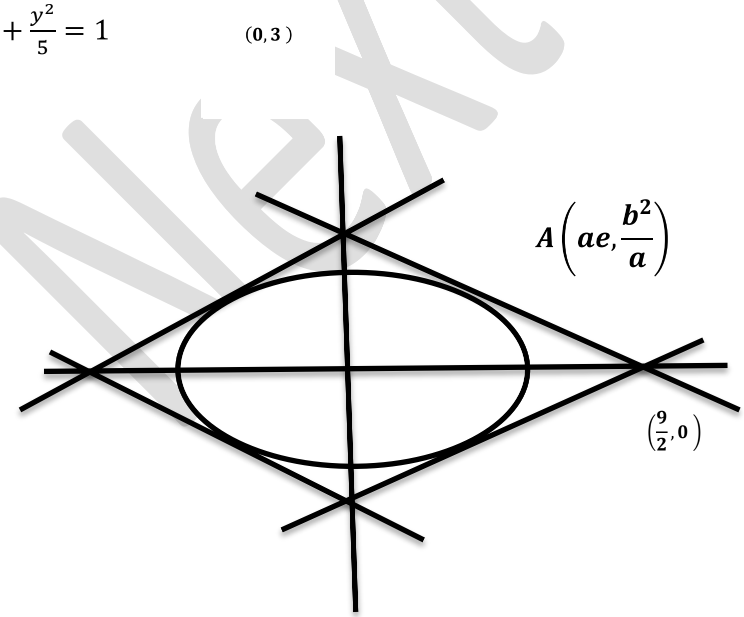
$C_1C_2 = R_1 + R_2$

Three common tangents.

81. Ans. 4.

Given ellipse is

$\frac{x^2}{9} + \frac{y^2}{5} = 1$



Equation of tangent at $A\left(ae, \frac{b^2}{a}\right)$

$\frac{xae}{9} + \frac{yb^2}{5a} = 1$

From above equation $a = 3$ and $b^2 = 5$

$\frac{xe \times 3}{9} + \frac{y \times 5}{5 \times 3} = 1$

$\frac{xe}{3} + \frac{y}{3} = 1$

From formula $b^2 = a^2(1 - e^2)$

$5 = 9(1 - e^2)$

$9e^2 = 4$

$e^2 = \frac{4}{9}$

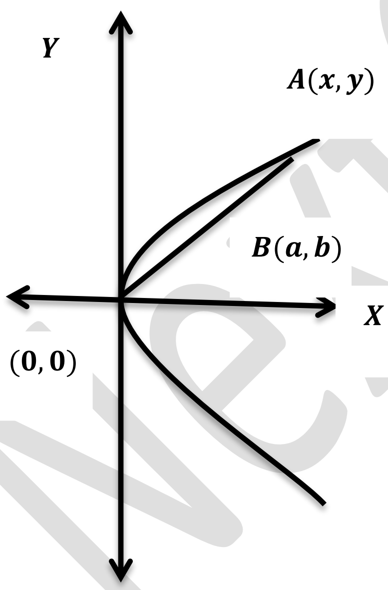
$e = \frac{2}{3}.$

$\frac{2 \times x}{3 \times 3} + \frac{y}{3} = 1$

$\frac{2x}{9} + \frac{y}{3} = 1$

This tangent have the following intercepts $\frac{9}{2}$ and 3.

Required area will be $= 4 \times \frac{1}{2} \times \frac{9}{2} \times 3 = 27.$



82. Ans. 4.

$\frac{x+0}{4} = a, \frac{y+0}{4} = b$

$x = 4a, y = 4b$

Given equation is $x^2 = 8y$
$(4a)^2 = 8(4b)$
$16a^2 = 32b$
$a^2 = 2b$.

83. Ans. 4.
Given line is $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = a$
Then
$x = 3a + 2$
$y = 4a - 1$
$z = 12a + 2$
Given equation is $x - y + z = 16$
$3a + 2 - 4a + 1 + 12a + 2 = 16$
$11a = 11$
$a = 1$.
Then $x = 5, y = 3, z = 14$
Distance between $(1,0,2)$ and $(5,3,14)$
$R = \sqrt{4^2 + 3^2 + 12^2} = \sqrt{16 + 9 + 14} = \sqrt{169} = 13$.

84. Ans. 3.
Given equations are $2x - 5y + z = 3$ and $x + y + 4z = 5$

Substitute $z = 0$ in the above equations
$2x - 5y = 3$ And $x + y = 5$
Solve for x and y
$x = 4$ And $y = 1$
Coordinates of the point $(4,1,0)$
The required equation parallel to the plane $x + 3y + 6z = 1$ is
$1(x - 4) + 3(y - 1) + 6(z - 0) = 0$
$x + 3y + 6z - 4 - 3 = 0$
$x + 3y + 6z = 7$.

85. Ans. 1.
Given (a×b) ×c=13|b||c|a
(c.a) b-(c.b) a=13|b||c|a

(c.a) b-|b||c| cos θa=13|b||c|a

(c.a) b=|b||c|(13+cosθ) a

(13+cosθ)=0
cosθ=-13
Sinθ=1-19=89=±223.

86. Ans. 1.

We want to find the probability that (at least) one of the boxes contains exactly 3 balls. Let the number of the boxes are 1, 2 and 3, and define Ei as the event that box I contains exactly 3 balls. We need to find the probability of the event $E_1 \cup E_2 \cup E_3$

$Pr(E_1 \cup E_2 \cup E_3) = Pr(E_1)+Pr(E_2)+Pr(E_3)-Pr(E_1 \cap E_2)-Pr(E_1 \cap E_3)-Pr(E_2 \cap E_3)+Pr(E_1 \cap E_2 \cap E_3)$

$=3Pr(E_1)-3Pr(E_1 \cap E_2)+Pr(E_1 \cap E_2 \cap E_3)$ (By symmetry)

$Pr(E_1) =$ (12 3) (29 312)

$Pr(E_1 \cap E_2) =$ (12 3) (9 3) (16 312)

$Pr(E_1 \cap E_2 \cap E_3) = 0$

Therefore, $Pr(E_1 \cup E_2 \cup E_3) = 3Pr(E_1)-3Pr(E_1 \cap E_2)+Pr(E_1 \cap E_2 \cap E_3)$

$=3\times$ (12 3) (29 312)-3(12 3) (9 3) (16 312)

87. Ans. 4.

Number of observations is 16 and the mean is 16.

$\overline{X} = \frac{\sum_{i=1}^{16} x_i}{16}$

$\sum_{i=1}^{16} x_i = 16 \times 16 = 256.$

$256 = x_1 + x_2 + x_3 + \cdots + x_{16}$

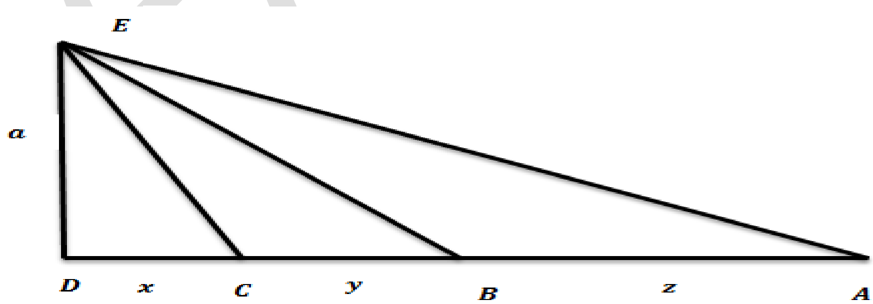
Given, if one of the observation valued 16 is deleted and three new observations valued 3,4 and 5 are added

$256 - 16 + 3 + 4 + 5 = x_1 + x_2 + x_3 + \cdots + x_{18}$

$x_1 + x_2 + x_3 + \cdots + x_{18} = 256 - 4 = 252$

New mean $= \frac{252}{18} = 14$

88. Ans. 1.



In triangle EDC

$cot\ 60^0 = \frac{1}{\sqrt{3}}$ implies

$x = \frac{a}{\sqrt{3}}$ …(i)

In triangle EDB

$cot\ 45^0 = 1$ implies

$\frac{x+y}{a} = 1$

$x + y = a \ldots$ (ii)

In triangle EDA

$cot\ 30^0 = \sqrt{3}$ implies

$\frac{x+y+z}{a} = \sqrt{3}$

$x + y + z = \sqrt{3}a\ldots$ (iii)

From (ii) and (iii)

$z + a = \sqrt{3}a$

$z = (\sqrt{3} - 1)a$

From (i) and (ii)

$\frac{a}{\sqrt{3}} + y = a$

$y = a\left(1 - \frac{1}{\sqrt{3}}\right)$

Required ratio $= AB: BC = z: y = (\sqrt{3} - 1)a: a\left(1 - \frac{1}{\sqrt{3}}\right) = (\sqrt{3} - 1): \frac{(\sqrt{3}-1)}{\sqrt{3}} = \sqrt{3}: 1$

89. Ans. 1.

Given equality is $tan^{-1}y = tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right)$

From the trigonometric formula $tan^{-1}A + tan^{-1}B = tan^{-1}\left(\frac{A+B}{1-A\times B}\right)$

Here $A = x$ and $B = 1 - x^2$

Then $tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{x+\frac{2x}{1-x^2}}{1-x\times\frac{2x}{1-x^2}}\right) = tan^{-1}\left(\frac{x-x^3+2x}{1-x^2-2x^2}\right) = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right).tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$= tan^{-1}y$

Implies $y = \frac{3x-x^3}{1-3x^2}$.

90. Ans. 4.

Table:

s^r is true iff both s and r are true.

svr is true if any one of them is true.

s	r	$\sim r$	$\sim r \wedge s$	$\sim s$	$\sim s\ v\ (\sim r \wedge s)$	s^r
T	T	F	F	F	F	T
T	F	T	T	F	T	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

Clearly from table equivalent to$s \wedge r$.